THE EXHAUSTIVE TREATISE ON SHADOWS

by

Abu al-Rayḥān Muḥammad b. Aḥmad al-Bīrūnī

Translation & Commentary

by

E. S. KENNEDY

Volume I

TRANSLATION

INSTITUTE for the HISTORY of ARABIC SCIENCE

University of Aleppo
Aleppo, Syria

1976

RESEARCH PUBLICATIONS

of the

INSTITUTE FOR THE HISTORY OF ARABIC SCIENCE

UNIVERSITY OF ALEPPO

THE EXHAUSTIVE TREATISE ON SHADOWS

by

Abu al-Rayḥān Muḥammad b. Aḥmad al-Bīrūnī

Translation & Commentary

by

E. S. KENNEDY

Volume I

TRANSLATION

INSTITUTE for the HISTORY of ARABIC SCIENCE

University of Aleppo

Aleppo, Syria

1976

to

Mary-Helen

PREFACE

These two companion volumes present, respectively, a translation of the Arabic text and commentary upon a work of the celebrated scientist of eleventh century Central Asia. The original, as its name indicates, is an extensive discussion of shadows, their nature, properties, and utilities, the author ranging about through the fields of optics, etymology, literature, religion, mathematics, and astronomy, as the main topic leads him.

As may be seen by consulting the table of contents, the book commences with a short apologia for taking up the subject at all. Bīrūnī then proceeds to define shadow, the phenomenon of night being interpretable as the most fundamental of all shadows. This leads into a discussion of the physical properties of shadow edges and rays of light admitted through pinholes. Several chapters follow in which four of the standard trigonometric functions are defined in terms of shadow, and their various relationships are worked out. It is natural next to describe the astrolabe and other instruments which employ the shadow functions.

The second half of the book gives solutions of a series of astronomical problems involving shadows: the noon shadow cast by a unit gnomon as a function of season and latitude, the determination of the local meridian by observations of shadows, timekeeping by means of shadows, daylight length as a function of season and latitude, and celestial distances involving shadows.

The times of two of the five Muslim daily prayers are defined in terms of shadows. Hence two chapters are devoted to this topic. The first

cites the traditions upon which the definitions are based; the second describes instruments for applying the resultant rules.

It is as a primary source for the history of the ancient and medieval exact sciences that Bīrūnī's *Shadows* is significant. The precursors of the tangent function he describes, particularly the primitive shadow tables for telling time, contribute to our knowledge of the prehistory of trigonometry. The Babylonian linear zigzag functions he passes along exhibit one of the very few direct connections between the astronomy of ancient Mesopotamia and that of early Islam. The meridian determination of Diodorus here preserved is the only solid information we have about the work of this first century B.C. Alexandrian.

The topics named above have already received some attention in the literature. Only a full translation of the text, however, can make available to historians of science generally the multitude of references to individuals, books, and theories, famous or obscure, extant or unknown, with which the book abounds. Individually insignificant, they are hitherto missing tesserae in the mosaic of history.

The translation is based on the Arabic text published in 1948 by the Osmania Oriental Publications Bureau and referred to here and in the sequel as the *Shadows*. (Short titles and abbreviations in italics are references to the bibliography which follows the commentary.) Page and line of the printed text are indicated on the margins of the translation, and the same system is used for cross-references to the text in the commentary, and for entries in the index.

The edition, in turn, is based upon the unique Patna MS 2468/36 preserved in the (Bankipore) Khuda Baksh Oriental Public Library. Thanks to the Honorary Secretary of this library a microfilm of the manuscript has been available, and the frontispiece of the translation reproduces the fine calligraphy of its title page. Beginnings of the manuscript folios are also indicated in the translation margins. Within the translation a double

diagonal stroke marks the place at which a new folio commences; the much more numerous single diagonal strokes denote beginnings of lines of the printed text. The variants between the two versions have been placed in footnotes.

Material enclosed within parentheses does not appear in the original, but has been added to clarify or improve the sense of the passage where it appears. Many phrases in the translation are stilted or awkward. In part this may be laid to the ineptitude of the translator. To an extent, however, they result from a desire to preserve some savor of the Arabic. Restorations to the text have been enclosed in square brackets. In general, both the original and the restoration are given in footnotes.

The paragraphing of the translation is that of the printed text; the manuscript has no paragraphs. The reader will notice that these subdivisions bear little relation to the subject matter. We nevertheless thought it best to preserve them.

A considerable portion of the Arabic text was omitted from the edition and printed without notice as part of a different book, listed as *Sinān* in the bibliography. The reason for this is that the manuscript, a single volume collection including many treatises in addition to the *Shadows*, suffered rebinding at some time in its history. Some of the folios, including this segment, were bound out of order. The intrusion, happily discovered by Professor A. S. Saidan, has been restored to its place in the translation. The gap commences at the middle of page 5 of the edition, the filler from the middle of page 34 of *Sinān*. It runs to the top of page 63, whereupon the edition picks up. In the translation and the index the excerpt from *Sinān* is distinguished by an *s* preceding the page number.

A less serious misplacement of the same sort occurs at 158:10 of the edition. The text from here to 160:4 should have been printed at 146:4. In the translation the missing passage has been restored to its proper place, but with the page and line numbers of the edition. The unfortunate reader who, working from the index, misses a reference, should consult this preface to locate the insertion.

The second volume, the commentary, has been set up with the same chapter organization as the text. The chapters, for ease of reference, have been further subdivided into short sections, numbered serially without regard to chapters. Associated with each section title is the portion of the text to which it refers. These portions are treated in the order in which they occur. References to the commentary in the index are given by section numbers in italics.

Since this publication does not include the Arabic text, already available, no Arabic-English glossary appears. The reader who encounters an unfamiliar word in the Arabic text will find its English equivalent at the appropriate page and line of the translation. He may then have recourse to the index if he is interested in additional occurrences of the same word or its synonyms. There is considerable need for a dictionary of medieval Arabic scientific terms; the sources for such a book should include much more than the *Shadows*.

In preparing the commentary an effort has been made to suit the needs of a particular category of reader, the historian of science who is mathematically and astronomically literate, but who is neither an orientalist nor a specialist in medieval astronomy. Even so, the choice of which topics to include and which not remains largely a matter of individual taste.

This is inevitable, and an apology would seem gratuitous. It is with diffidence, however, that a translator who lays no claim to being an Arabist makes public an attempt to English the *Shadows*. By rights he should be simultaneously competent also in Islamic studies, oriental poetry, and classical philosophy. The rueful words of Sachau are recalled in the preface to the *Chronology*: "The work of generations will be required to do full justice to Albiruni". Here is a beginning; let the reader correct the errors he finds.

The translation was made possible by a succession of grants from the National Science Foundation to the American University of Beirut, and by appointments to Brown University and to the Institute

for Advanced Study. It is always a pleasure to thank friends and colleagues for help: first, as ever, Professor O. Neugebauer for counsel extending far beyond this particular work to span the better part of a lifetime. Specific contributions made by Professor David Pingree have sometimes been indicated in the commentary, within parentheses and followed by his initials, but his assistance includes any sections involved with the Sanskrit sources. Professor Adnan Ifram read through the entire translation in its preliminary form and rescued the translator from all manner of blunders. Professor Ihsan Abbas has taken time to elucidate troublesome questions of Arabic poetry and Muslim tradition. Professor Jibrail Jabbur, the late Professor Salwa Nassar, Professors Kamal Salibi, Majid Fakhri, Fuad Tarazi, Dr. George Saliba, and Mr. Taysir Salihi have all assisted. Heartfelt gratitude to them, with no thought of their being implicated in errors committed by the undersigned.

Copy for photo-offset printing was turned out in Beirut simultaeously with the development of the Lebanese civil war. The concomitant difficulties provide a blanket excuse to cover the manifold shortcomings of the result (the bizarre format of this page, for example). Moreover, the milieu in some ways appropriately resembled that of the wars of Sulṭān Mahmud, and the vicissitudes under which al-Bīrūnī brought forth the original of this work.

The very demanding typing was commenced by Mrs. Kawthar Shomar, thence, during her temporary absence, taken over by Mrs. Annie Aroyan. Any elegance the edition may claim is owing to the technical advice of Mr. Zahi Khuri.

Finally, to the Director, and all connected with the Institute for the History of Arabic Science, for indispensible support, and to Mr. Muwaffaq Ghannam, for seeing the project through the press, warm thanks.

E.S.K.

CONTENTS

(Bibliography and indices are at the end of Volume 2.)

TRANSCRIPTION OF ARABIC LETTERS

ON THE FIGURES

In the translation the figures retain the numbers and lettering of the text. Where a sphere is represented, the figure has been redrawn in orthogonal projection to facilitate understanding of the spatial relations. In the body of the translation the standard conventions have been used for the transliteration of Arabic words into Latin characters. However, individual letters on the figures have been transcribed as shown below. The scheme is a slight extension of the one proposed in the *JAOS*, vol. 82(1962), p.204, and is employed for the reasons there set forth.

A	ا	*M*	م
B	ب	*N*	ن
C	ص	*O*	ع
D	د	*Q*	ق
E	ه	*S*	س
F	ف	*T*	ط
G	ج	*W*	و
H	ح	*X*	ش
K	ك	*Y*	ى
L	ل	*Z*	ز
	θ ث		

TRANSLATION

OF THE

TREATISE ON SHADOWS

(DEDICATION) f.194b

In the name of God, the Merciful, the 3:1
Compassionate:

A discussion of visual perception and the 2
nature of the cone/ existent between the instrument 3
of sight and the object seen, the source[1] of which
(the cone) is at a distance (from the object, and
which) entails/ the geometry of optics in its 4
different (forms), whether it is due to the ray
which emanates from the beholder unto/ what he 5
beholds, or due to the ray resulting from the
images of objects and their colors, and its
impression/ on the vitreous humor of the eye, 6
(such a discussion) is a philosophical matter
pertinent to psychological investigation and to/
abstract speculation entrusted to those talented 7
in them (i.e., these fields).

However, as to investigation of the actual 8
light and what is connected with it, and what (is
connected with) its absence, called/ shadow in 9
general and shadow specifically, it pertains to
the types of mathematical science which/
[facilitate][2] (the solution) of problems of any- 10
one who resorts to religion, depending on the
ways of/ evident truth, like the Shaykh Abū 11
al-Ḥasan Musāfir b. al-Ḥasan, who is embellished
by these characteristics./ Verily he is famous 12
for his burning desire for the knowledge of the
times of prayer and his extreme devotion/ to 13
whatever instruments are referred to for their

---

[1] Text اصبعه ; MS is not clear; read منبعه .
[2] Text اعراض ; read اغراض .

determination, (as a result) of his care for the happiness of being rewarded (in the afterlife) after having/ been given by God the happiness of the first life, which makes him seek the virtuous in the two happinesses. 3:13 14

I will be discussing of that, what will suffice for the untying of this knot/ and the acquisition by it of the advantages of being praised. For there is no one in the world who does not naturally attempt/ to make permanent his kind who does not strive to immortalize his fame. So by neccesity the wise man satisfies himself with/ the remnant called his body, to be respected in spite of the passage of nights and days after him. And because/ the good is loved for itself, since even the wicked desire it (the good) for themselves, although they may stray/ from it, the desired (type of) sayings is the best (kind), and of the/ enduring fame the good and the beautiful. So, blessed is he for whom the blessing of God, be He exalted!, endures, by the endurance of thankfulness/ and the choice of the most praiseworthy of affairs. I request of God for the Shaykh divine success whereby he may/ be in the forefront in the attainment of his desire, and for myself (I pray for) striving to approach satisfying Him and to maintain (my) [enjoyment][1]/ of His benevolence, by which the people rejoice. (Verily) He is the Master of Destiny for making accessible things of great importance, by His grace,/ and the extent of His generosity. 4:1 2 3 4 5 6 7 8 9 10 11

[1]Text الامتناع ; read الامناع as in the MS.

(TEXT TABLE OF CONTENTS)

This is a list of the chapters of the discourse, into which we will plunge[1] in order to facilitate/ the extraction of what is desired from it. 4:12 13

[1]Text: نخوص which makes no sense; MS has نخوض .

This is the end of f.194b. The MS was evidently bound with the folios in disorder; the folio which should have followed is 125. The displaced passage, as discovered by Professor A.S. Saidan, was inadvertently printed as part of Ibn *Sinān*, beginning in the middle of line 8, p.34, Treatise 3. A translation of the missing section follows. Page numbers from *Sinān* are preceded by an s.

f. 125a s34:8

from the Other.

12. On Tables Containing Shadows, Exclusive of their Computation, and How/ to Obtain Them (the functions) from Them (the tables) Until the End, and [Considerations][1] Involving Them.

13. On Fixing the Kinds of Shadows on the Astrolabe So that They Will Be/ Useful for What Follows.

14. On Fixing the Ladder Shadow on the Astrolabe.

15. On Shadows/ Measured on Inclined Planes or on Other [Things][2].

16. On the Determination of the Noon Shadow for any Assumed Day.

17. On the Equinoctial Shadow for [Any Locality][3].

18. On the Correction of the Meridian Direction by Two Shadows or by Two Equal Azimuths.

19. On the Correction of the [Meridian][4] Line.

20. On the Extraction of the Meridian Line [by] the Use of Three Successive

[1]Text and MS; حيالا read خيالا ?
[2]Text: اوعلى غيرمقيسة , but cf.81:2 اوعلى غيرها .
[3]Text: في يد ; MS: في بلد read في كل بلد as in 94:14.
[4]Text: نصف خط النهار ; read خط نصف النهار as in the MS.

[Shadows][1].

s35:1

[21][2]. On the Extraction of [the Meridian Line][3] by Any one Single [Measurement] Whatsoever.

22. On the Amounts of the Day and the Night and the [Differences][4] of the Ascensions.

23. On the Determination of What Is Past and What Remains of Day(light) by (Use of) the Shadow.

24. On the Determination of the Azimuth and Its Ascension.

25. On the Recital of the Opinions of the *Imāms* Regarding the Time of Prayers, and What Is Resorted to/ in Determining Them.

26. On the Establishment of the Lines for the Times of Prayer and the Hours on/ Instruments.

27. On the Use of the Shadow in the Quadrilateral[5] (i.e., Menelaos') Theorem and in Astronomical Computation.

28. On the Determination of [Terrestrial][6] Distances and the Heights of Mountains/ by (the Use of) Shadows.

29. On the Determination of Celestial Distances Which Involve Shadows.

30. On the Explanation of Things Connected with the Shadow and Not Resembling What Has Preceded.

[1]Text: ثلاثة اخلال ; read بثلاثة اظلال as in the MS.
[2]Text: ي ; read كا as in the MS.
[3]Text: مقيسة ; read خط نصف النهاربقيسه as in 120;2.
[4]Text: فصول ; read فضول .
[5]Restore to الشكل القطاع omitted in text and MS; cf.194:10
[6]Text: الارضية والسماوية ; cf.202:7.

(AL-BĪRŪNĪ'S PREFACE)

I say firstly, that the subject of this s35:14
investigation can hardly be comprehended except
after/ encompassing (knowledge of) the constitution 15
of the universe according to what is shown by
demonstration, excluding what/ the various groups 16
of people apply[1] to it of what they have heard from
their ancestors, as well as recourse from the sects
to/ their beliefs, and (also) after (attaining) the 17
capability of dealing with its varying situations,
in which one cannot dispense/ with arithmetic and 18
deep investigation of it by geometry.

Verily, (even) he who has studied much in 19
the sacred books may not be separated/ from the s36:1
mass of the common people, nor from their conviction[2]
that this art is contradictory to religion, contrary
to divine (Muslim) law; that it is/ a forbidden 2
pursuit, and an abrogated and forsaken practise.
Nothing impells him to this belief but/ his ignorance 3
of what impugns religion so that he might (properly)
support it, his revulsion from the unfamiliar which
he inherits from/ [his likes][3] before him, and his 4
inability to distinguish what is (truly impugning
to religion) from what is not./ Thus, if he learns 5
that a matter is as he thinks, he does not accept
what is traditionally said about it —/ an excellent 6
thing, should he prove to be unrelying on tradition
in what he believes or thinks. And if he is shown
that/arithmetic and geometry are impossible to under- 7
stand unless one proceeds systematically from first
principles, unlike other sciences in which he may
be acquainted with/ something of their middle (parts) 8

---

[1] Text يطتق ; read تطبق as in the MS.
[2] Text اعهارها ; read اعتبارها .
[3] Text امثال ; MS امثاله .

or their ends without knowledge/ of their s36:9
beginnings, he thinks that this is intended to
[turn him away][1] from his appreciation/ and to 10
confuse him. This, he imagines, is similar to
the ignorance into which (non-initiate) members of/
(secret) sects (are led) with regard to the 11
doctrines of their sects until they had taken
the oaths, entered into the covenants, and made
a long practise/ and training. This adds to 12
his revulsion, so that the stopping of his ears
with his fingers[2] becomes his most potent/
recourse, and the raising of his voice in s36:13
shouts his most powerful equipment. (Now,
suppose that) he should desire to recite some
special verses of poetry and that he should/
seek them from the anthologies of Dīk al-Jinn, 14
Abū Nuwās, Abū Ḥukayma,/ and Ibn Ḥajjāj. These 15
(anthologies) contain silliness to make the soul
of the wise man recoil; impiety/ to exceed all 16
unbelief, and (a wide selection of) lies used as
poetic ornament. But he will not be able to
tell/ how bad or how good those verses are until 17
he hears them with his own ears; (and it is only
then that) he will know what is good in order to
take it, and what is bad in order to avoid it.
However,/ he does not know that the extent of 18
understanding among the common people of a problem
of the minutiae of theology concerning the bases
of/ canon law or the like is as the extent of his 19
understanding, if (indeed) he understands it, even
if it is encompassed at all, of a question of
medium order/ in geometry. And verily, both s37:1
understandings, if they are approached by
systematic learning,/ questions in both of the 2
two arts are attained, and they are realized in f.125b
an//elegant manner, and the acquisition pulls the/
curtain of doubt from between them and the truths 3

---

[1] Text رواعا ; read رجوعا .
[2] The figure of speech is from the Qur'ān, 2:19.

of knowledge concerning both. Then, if he knows s37:3
that prayer is the/ buttress of religion, and 4
that its perfection is restricted to (its
observance) at its (proper) time and facing
in the proper direction/ for it, and that both 5
matters are connected with astronomy and a due
amount/ of geometry; and almsgiving follows them, 6
and [inheritances][1] there being no escape from
them, just as there is no escape from/ buying and 7
selling as a means of subsistence, in the Muslim
law, and (since) all of them require/ arithmetic, 8
either in the lowest degree, in imitation of the
method(s) of the computers, or else/ at its highest 9
level, it being the deep investigation of geometry,
then people accuse him of error/ and denial and 10
claim that he is not pious because of these two
arts, but how so? For he is obliged/ to apply 11
the two in almsgiving for the manufacture of weights
and measures, and in charity/ the making of
(standard) units, and for the holy war numerous 12
manufactures and various instruments/ of steel, 13
welded with violent power (are) necessary.

The learned in religion who are deeply 14
versed in science know that Muslim law does not
forbid/ anything of what the partisans of the craft 15
of astronomy (concern themselves with) except the
lunar crescent. For it is placed/ on visibility 16
without the use of computation, and the reason for
this is apparent to anyone who has a thorough
grasp of how to/ obtain the arc of visibility at 17
(the time of) the fast by the crescent operation.
For when he acts impartially/ he becomes aware of 18
the fact that visibility with the eye depends for
certainty upon the result of computation at the
time when/ the entire amount of this angle is 19
approached, since the peoples' operation for
visibility/ is other than what is taken for it in s38:1
Muslim law, the toil involved being great, and
the benefit their ability to/ determine the position 2

---

[1]Text المواريث ; read المواريث as in the MS.

of the crescent in azimuth and altitude so that s38:2
the observers may look for it/ with the *abṣār*, 3
and (thus) dispense with (the necessity of)
ranging in/ sight over a region of the sky, around/ 4
the perpendicular of the twilight, and having
length and width, lest that ranging divert them
from catching it/ until it vanishes. 5

As for the few whom the revealed word s38:6
(the Qur'ān) praises, whose assiduous working in
the fear of God, be He exalted!,/ distracts from 7
venality, they are the ones who do not establish
a judgment before deep investigation, and who are
not obstinate in opposing a situation whose/ truth 8
is evident, and who commit nothing against Islam,
nor attack the Qur'ān, nor pretend that there are
differences/ as to essentials. Those people are 9
between two choices, either to obtain assistance
in any art/ from its practitioners, it being a 10
thing commanded, or else to divert their endeavors
to finding the desired truth[1]/ by lavishing toil 11
upon it, for the sake of being innocent of the
stain of mimicry and ignorance,/ God set us among 12
them, by His Grace!

---

[1]Text استيقان ; MS استيعاب .

THE FIRST CHAPTER s38:13

ON (THE FACT) THAT THE PRIME MOTION OF 14
THE HEAVENS IN THE WESTWARD/ DIRECTION 15
IS NECESSARY FOR THIS TOPIC

If it were not for the bodies perceived 16
in the heavens, it would not have been known that
there is motion in the heights;/ and if there were 17
no upper motion, no direction would be known on
the horizon except by an arbitrary setting. If
the directions were/ specified by a setting at one 18
of the terrestrial localities, the identification[1]
would not be/ exact in that region. So the risings 19
of the two luminaries and the various stars,/
even if the horizon is not bisected by their s39:1
settings, but is divided into at most two/ unequal 2
parts, verily the two directions, north and south,
are of necessity between each rising point/ and 3
its corresponding setting (point), hence they are
evident by the prime motion,/ from which are the 4
risings and settings. But if the direction of north
is ascertained, the pole and the rotational motion/ 5
are of the class of mutually related (things) of
which neither one precedes/ the other, just as the 6
determination of the direction of north, together
with its opposite, I mean south, is of the class of/
mutually related things also. In addition to this 7
the occurrence of this motion is of significance

[1]Text has تعيين ; MS تتعين .

for the determination of position[1]/ in a level s39:8
desert locality whose parts and regions resemble
one another, either in the night/ or in the day- 9
time. Verily its heaven [darkens][2] until it
becomes dark equally throughout its air. So
undoubtedly/ the time cannot be ascertained, at 10
night or in the day, nor can any one of the four
directions (be determined) without the others,/
and that is because of the lack of guiding 11
indications for them. Even if a person finds
by chance marks of the directions fixing/ them 12
they do not agree with other works taken as valid
for his station, and for (individuals) under the
same circumstances (but)/ in a different locality, 13
except rarely by chance, because it is located by
guesswork without a law/ to be referred to or a 14
sound base which can be depended upon. By this
motion God, be He exalted!, recalls to His
creation His benefits in His saying[3],

> Say: See ye? If God 15
> Were to make the night
> Perpetual over you until/ the day 16
> Of judgment, what god
> Is there other than God,
> Who can give you enlightenment?
> Will ye not then hearken?

And in His, be He exalted!, saying[4],

> Say; See ye? If God 17
> Were to make the day
> Perpetual over you until the day
> Of Judgment, what god/
> Is there other than God, 18
> Who can give you a night

[1]Text المنية من سنته ; MS المنتبة من سنته .
[2]Text اعلمه ; read اعتمت .
[3]Qur'ān 28;71.
[4]Qur'ān 28:72.

[In which][1] ye can rest? s39:18
Will ye not then see?

That is, that these two/ situations will 19
not occur until after the decline of this motion,
and (that of) perceived bodies which move by it. s40:1
Also, verily time is the extension between 2
two assumed instants, the two being two times/ of 3
two known states, and because of the existence of
these[2] two situations, one after the existence of
the other,/ the extent (of time) between the two 4
may include length or shortness, and (whatever)
situations which may exist in it/ in succession 5
capable of having smallness and largeness. Verily,
it is like the distance between two endpoints/
and distances cannot be controlled accurately 6
except by motion, and those of them which are
controllable are the equable (i.e. constant speed
motions) excluding/ the disturbed, different 7
(speed motions). Equal motions have become the
measuring units of time, indicating/ that by 8
clocks operating by the motion of water or sand
or various (varieties) of [seeds][3],/ or things 9
resembling them. Indeed the object in making them
is uniform motion, even though they are not
equivalent/ except approximately to the senses. 10
And because equal motions are midway between
slowness/ and speed, and slowness is bounded on 11
its two sides by [stopping][4] and speed, which
(latter) is essentially unbounded/ as to the 12
amount at which it stops, except in actuality.
As for (the applied) force, it is subject to
increase/ just as a number (increases) in the 13
direction of its growth. So there is no speed
(concerning which) we cannot imagine that behind
it there is no speed greater than it./ So the 14

[1]Text فيد ; read فيه .
[2]Text ذينك ; MS ذانك .
[3]Text البزور ; read البذور .
[4]Text الكون ; MS السكون .

fastest of existent motions is the prime s40:14
(motion), by which are the night and the day
(made), and verily/ that is established by the 15
magnitude of the extremity of what is moved by
it, and the magnitude of the extremity of what
is after it, and by it is found/ the noon of 16
the parts (or units) of time, I mean the day.
So this motion has been made the cubit (i.e. the
unit) for time (measurement),/ and the evalu- 17
ation of it is by its uniformity and its speed.
As for speed, it is unnecessary (that it be
discussed here), but it/ was mentioned by reason 18
of its being the extremity[1] of existence. As
for uniform (motion), it is necessary (for our
discussion) and since/ the matter is thus, it is 19
incumbent upon us in what we propose to give our
attention to the operations/ by [which][2] direc- s41:1
tions and azimuths are determined, and to fix
thereby instants in time.

[1]Text لنهاية ; MS النهاية .
[2]Text بها ; MS التي بها .

THE SECOND CHAPTER s41:2

ON THE EXPLANATION OF LIGHT AND DARKNESS 3

LUMINOSITY AND SHADOW

The brilliant (one), in reality, of the 4
bodies perceived as luminous, is the sun, which is/
self-luminous, illuminating others than itself by 5
the ray issuing from it in all directions,
penetrating/ transparent objects rectilinearly 6
until it impinges upon an opaque body. And the
state of a body which interdicts/ transparency is 7
that a ray of light which is confronted by it does
not penetrate it, but is turned back, being/
reflected from it depending on the smoothness of 8
its surface which the ray encounters. If it is/
extremely well polished and evenly disposed (i.e. 9
plane) as to its parts, it is [not][1] perceived as
though it were (the object) upon which the light
is falling,/ but it is perceived as from where it 10
has been reflected. But if it is not a uniform
polished (plane surface),/ the reflection from it 11
will be weak and the light will be seen upon that
surface as stable, and/behind it will be dark,// 12
contrary to (the situation) in the direction of f. 126b
the illumination because of the absence of light
at it, and that absence, provided it is restricted/
to a place not interfered with by the boundaries 13
illuminated around it, and its image is not/
perceived by the eye, except to a slight degree, 14
then it is called the shadow. This is the
opposite of what is called metaphorically the sun,/

---

[1]Text ثم ; MS لم.

I mean the brightness. That is similar to the s41:15
shadows of things which/ fall upon the face of 16
the earth, or walls. So the brightness, I mean
the illuminated places which go beyond/ so as 17
to receive the light, is perceived along its
edges, totally/or partially. However, if the 18
ray is not perceived from one of its sides, and
the quantity (of shadow) is increased because of/
the increased extension of its (the object's) 19
limits in such manner that sight is lost in it,
and it (sight) does not perform its function, it
(the shadow) is called darkness and absolute absence
of light,/ like the situation at night or on a s42:1
cloudy day. So the name of shadow then vanishes,
just as (the ability) to [perceive][1]/ its 2
extremities also vanishes.

Al-ẓill (shadow) in the speech of the 3
Arabs, is a covering from the sun, and from it is
darkness, and hence/ the blackness of night[2] is 4
called a *ẓill*, and because of the contiguity of
ẓill and light and the following of one by/ 5
the other they call the bounded *ẓill*, surrounded
by (the edge) of the sunlight, a follower. As one
of the (Banū) Hudhayl said/ in the poem, 6

> ... The coming of the sand- 7
> grouse to water when the follower
> (i.e. her shadow) contracts.

Abū Laylā said concerning it that here the 8
night (is intended), as if he had said, "He comes
to the water at daybreak before/ anyone". But we 9
do not see anything preventing his coming to drink
at noon, because the characterization/ of the 10
shadow as becoming shorter is appropriate to it,
and so he comes to the water (to drink) when no
one else (leaves) [his shelter][3]./ However, it 11
was said concerning the shortening that it was the

---

[1]Text الادارك ; MS الادراك.
[2]Text لليل ; MS الليل.
[3]Text لاختنامهم ; MS لاحسابهم ; read لالتنائهم.

arrival of the shadow at the base of the stick (i.e., gnomon). s42:11

Verily, Ru'ba makes a distinction in the nomenclature between what declines of it and what is fixed, and he said that/ *ẓill* is the name attached to a place which presently has no blackness in it which lasts, nor is there sun(light) on/ its place. So it is attributed exclusively to the position on which the sun(light) was and then it left it/ into the shadow (*al-fay'*) because *al-fay'* is the declining and the return. Hence *al-ẓill* is more general (and) *al-fay'* more particular. Thus/ every *fay'* is a *ẓill*, but not vice versa, that every *ẓill* should be a *fay'*. But what was explained concerning/ *al-fay'* does not prevent its being present before noon.

Ru'ba said concerning these details that *al-ẓill* is what is formed by the sun, while *al-fay'*/ is what impedes the sun, meaning by the "sun" s43:1 the place where its light falls upon the earth./ What is well-known as to that is that the Arabs call the *ẓill* after noon *fay'* because of its inclining from/ the western side to the eastern side entering, and its return increasing (to cover) what was/ before in sunlight. This rule of theirs implies the abandoning of this name for it at high noon./ However, they transgress it and call it *ẓill* at noon.

Some of them said that it is called the *ṭibāq al-khuff* (the fitting of the camel's foot in the depression it makes), but that is not permissible except with its/ vanishing at noon, and the sun's being at the cupola (i.e., the zenith). It is as though the details are the work of/ industrious grammarians (who were) not Arabs. So they mixed things up, and failed to make that/ definition, but decided the matter in any way it happened in order to force the legalists/ to justify it thus.

Abū Dhu'ayb said,

> And I sit down in its *aẓlāl*
> (pl. of *ẓill*) in the
> afternoons.

And Dhū al-Rumma said, s43:13

> If the *ẓill* was changed in the late afternoon you will see it as a
> Ḥanafī, but in the high forenoon as a Baṣrī.

This is the description of the chameleon, which always faces the sun, as Abū al-Najm said,

> You will see the chameleon in that place bowing in entreaty
> Like [a pagan][1] to the sun, then kneeling.

But there is nothing strange in this, for the leaves of the trees by their natural disposition also turn about/ with it (the sun), but Dhū al-Rumma did not say, "If the *fay'* was changed in the late afternoon./

And if it is said that the evening changes the *ẓill* into a *fay'*, another said,

> (There was) a town, its voices silent, s44:1
> Its *afyā'* (pl. of *fay'*) diminishing in the morning sunlight.

It does not say, "its [*aẓlāl*][2] (pl. of *ẓill*) in the morning sun,[3] as though *aẓlāl* is not valid for the (morning) sunlight, and/ it is not annulled except at noontime, if its disappearance is possible at all. So if *fay'*// in the sun- f. 127a shine/ is permissible it is what we said, and if that is for the sake of the rhyme it confirms it, this being/ the custom of the literati and the poets in subjects like this, so that the reader of their/ sayings is compelled to extract their [possibilities][4] from them. In the book *Diwān*

---

[1]Text كوافر ; read كافر .
[2]Text اضلالها ; read اظلالها .
[3]Text واذ ; read زاد .
[4]Text وجود ; read وجوه .

al-adab (it says) that kindness/ is the *ẓill* of a ray of the sun in the morning and in the evening. s44:7

Someone also said that the *ẓill* of the sun is when it first begins to get hot, as it is said that the *ẓill* of winter/ is at the first of what begins, but that is like something he does not understand, the *ẓill* being for the shadow-casting body, not for the ray./ Thus spoke al-Khalīʿ al-Shāmī,

> Look at the *ẓill* as it reaches
> its extreme (length).
> It commences to decrease as
> time lengthens.

But it is evident that the greatest length of the shadow extended along the ground will be at/ (sun)rise and sunset, and at one of its extremes it begins to contract and decrease, and at/ the other arrives at its completion by the dropping away of the ray from both its sides. This would be acceptable/ if the shadow were lengthening at noon, but (the situation) is not thus except in (the case of) the shadow of/ a gnomon perpendicular to a wall whose base is along the meridian (line), it being what is called/ the reversed shadow and not, by God!, what was meant by al-Khalīʿ. He had in mind the saying of the first:

> Whenever an affair reaches
> completion its decline
> approaches.
> A falling off (*zawāl*) occurs
> when it is said to be
> complete.

So this (meaning of) *zawāl* was carried over to the decline of the sun (*zawāl al-shams*), and the extremal was changed into the shadow. So s45:1 that was what he said, and perhaps someone explained it by having heard that/ a lunar eclipse is from the shadow of the earth and he intended it to be

(relating) to an eclipse. (The latter) reaches s45:2 its (maximum) magnitude upon/ the arrival of the moon at its nearest point of passage to the axis of the shadow cone, which is its maximum length, (then)/ it begins to decrease, and the return is little by little until clearance. Or, since[1] night is/ nothing but our being in the earth's shadow, whose axis, if it is erected near us/ at midnight, is afterwards depressed until daybreak and morning and twilight, but all this is/ far from the mind of the author of the poem.

If you reflect upon revelation (in the Qur'ān), you will find it to be according to what we explained, and that is that pious and devoted people,/ who are to be blessed with deliverance from change, and their time from passing away, the sun can then be dispensed with/ for [enumerating?][2] the periods of time by the motion which makes apparent the traces of growth and existence/ in (various) places. So their place is characterized by the shadow's being (permanently) [extended][3] in time and space. As for/ time, since it is invariant with respect to sunshine it follows it, and as for space (*or* the place) since it is infinite/ with respect to a shadow in it, and what is behind it will be the sun, but (that place) is a shadow, its shade long-lasting, with no sun/ in it to obliterate it, and no hot wind (*simūm*) to decrease it and spoil it, as though it is free from anything of this description, it being/ the cold, as His saying[4], be He exalted!,

> They will see there neither
> The sun nor *zamharīr*,

---

[1]Text اذن ; MS اذ.
[2]Text العارة ; MS العازّة.
[3]Text المدور ; MS الممدود.
[4]Qur'ān, 76:13.

that is, (neither) heat (nor) cold./ It is s45:16
what the Muslims mean in explaining (it as),
"Those (women) whose curtains are shortened,
who (nevertheless) never see sun/ nor *zamharīr*". 17
And (this is so, even) though some of them
artificially claim that by *zamharīr* the moon
is intended. This is/ the case either because 18
they assume that the two luminaries are always
to be mentioned together, or else because they
attribute cold to the moon/ on account of the 19
attribution of heat to the sun. This is the
opinion of the Indians, who do not know that/
the moon heats up without the heat of the sun, s46:1
until it is the cause of the ebb and flow (of
the tides) and other/ events occuring with 2
moist things.

However, as for the people worthy of 3
punishment, the shadow they know (in hell) is
characterised as smoke (*yaḥmūm*) because the
utility of shadow is/ relief from the distress 4
of heat and the *simūm* (a hot wind), and if it
(the shadow) were other than cool and not
pleasant it would/ increase the painful torment, 5
like the distress present at the strata of the
sky which [takes][1]/ the breath (*or* souls) away 6
and which [chokes][2]. Verily the [radiance][3] of
the sun and its heat are more bearable (than it)./
Also their light is of burning heat and their 7
shadow is of smoke, and hence this/ shadow will 8
not be extended[4] but its ends are shaped by
limits, because smoke is from/ the tongue of the 9
[flame][5], restricted to one place and not another.
Hence (this variety of) shade is described/ as 10
portions of (hell)fire not bringing relief from

---

[1]Text الاحد ; read الاخذ as in the MS.
[2]Text الحايق ; read الخانق .
[3]Text صمع ; read ضح .
[4]In Qur'ān 56:30 the shadow in paradise is described
with the same adjective, *mamdūd*, extended.
[5]Text النهار ; read النار .

the flames. Verily, being exposed to smoke is s46:10
more distressing than being burned/ or heated. 11
It may well be that the portions (of flame) are
descriptive of its form, just as figuratively
the flame may be restricted to/ its sparks.// 12
Or it may be that the portions are the f. 127b
directions in which one sees to/ front, and 13
right, and left, because behind, even though
it is a part of them, is not different from
them as to/ detestable attributes. So it 14
cannot be perceived or seen without turning
around./ And the visible portions, and the 15
remaining directions, above and below, are like
the three mentioned (above)/ as to detestability, 16
as God, be He exalted!, said of them[1],

> (For them) there is
> Hell, as a couch
> (Below) and folds and folds
> Of covering above.

They (above and below) were not mentioned 17
as being among the portions because they are
equivalent to behind in not being perceived/
before changing positions. 18

Verily Abū Muslim al-Isfahānī said that, 19
"God, be He exalted!, named the fire a shadow/
because it surrounds the punished (people)". s47:1
But this is outside the customary understanding,
and especially (taken together) with His saying,
be He exalted![2]/

> (... no shade 2
> Of coolness, and is
> Of) no use against
> The fierce blaze.

The shadow is surrounded but not
[surrounding][3]. Then he describes the portions

---

[1]Qur'ān, 7:41.
[2]Qur'ān, 77:31. تعالى is not in the MS.
[3]Text محبطا ; read محيطا .

as having/ three attributes. One of them is that it is other than shady, and the second (is) "no use against the fierce blaze",/ and the third (is) to cast down sparks. And if one meditates on the verse he will find two of the attributes/ of the shadow in the masculine form, and the third is attributed to fire in the feminine, and if it were permissible/ to attribute the name shadow to fire from the linguistic point of view, it would be permissible that its portions are/ the three triangles, or the angles at their bases, and the fishbone shape is called fiery, so/ let us return to what we were dealing with.

We say that it is known that the air which fills up the heavens is transparent, and so the light/ in it is not perceived, and the earth in the middle of it is a surrounded, non-transparent body. So the part of it opposite to/ the luminary is necessarily shone upon and illuminated, and the part of it not opposite it is dark, and it is/ well known that it conceals some of the air because of its circular shape and the bent part/ even if it were, and their ratios to what is illuminated, I mean as a cylinder, are evaluated at equality, and as a cone/ having bases differently placed (the ratios are) different (from one). But the light of the sun includes/ the sides of the whole earth composing a cone opposite the sun in/ a pine-cone shape, the axis of which is the diameter passing through the two centers of the sun, along the pine-cone shape,/ and the earth. It becomes fine at its distance from the earth until it disappears above the moon. That is/ because of the excess of the size of the illuminating sun over the size of the shadow-casting earth, and this/ shadow becomes an eclipse for the moon by its traversing it (the shadow) in travelling. The shadow (*ẓill*) of the earth is called its shadows (*ẓilāl*), not(?) from/ an imaginary point of view, (but actually), and especially during lunar eclipses, for the observer/ imagines it superbly from the earth and he pictures its limits of illumination as surrounding

it, or else/ the night alone is present and there is nothing but it and its passage by us. But it (the night) is not called, in spite of that, in reality/ a [concealing][1] shadow because of the distance of the extremities and their absence from the senses.

It is said as to the meaning of God's saying, be He exalted!,

> Have you never seen your
> Lord, how He has extended
> the shadow?[2]

It is the night and its extended darkness, and that is permissible, because the revelation (is so explained) according to the customs current among/ the Arabs. And if its meaning is the night, then its object would be either the general darkness at the/ emptiness of the sky, while[3] the sun is imagined to be nonexistent, even after the sun was created/ when it was illuminated, distinguishing the darkness, and the more sharply after it is not restricted to one place on it and no/ other, or else the object of it (the shadow in the quotation) is the earth's shadow, which is the night with us,/ when passing by us. Otherwise, if it were the prime motion (which is) western, the shadow would remain in a fixed place,/ because of the sun's staying. But this motion, as it rotates everything,/ so the sun enters the [darkness][4] of the earth, and nothing of its traces remains except a very little in/ the west at sunrise. His saying, be He exalted!,

> Then we seized(?) it for
> ourselves in a light
> grasp(?)[5]

---

[1]Text has مشاره ; read ساره .
[2]Qur'ān, 25:45.
[3]Text منذ ; MS عند .
[4]Text فالشمس دخلت الظله ; MS بالشمس وعلت الظلمه .
[5]Qur'ān, 25:46.

He means by it motion, because He, be He exalted!, s48:15
is not subject to where or when, being superior
to/ time and space, but the word(s) "for ourselves" 16
(*ilaynā*) occur because of what is moving in it/ by 17
the desire for it. It is possible that the meaning
of the verse is the shadows of gnomons which are
indicated by/ // the sun by surrounding them at f. 128a
their edges, and positions, and it was made as
moving,/ and the motion was attributed to it, even 19
though it (the shadow) is incorporeal, because of
the fact that the sun, being/the author of its s49:1
increase and decrease, moves its edges and carries
it, and it is known (that) from[1] the stillness of
the shadow/ is the disturbance of the world./ 2
It may be that His saying, be He exalted! 3

> Then we grasped it for
> ourselves[2],

points to/ noon, and this is indicated by His 4
saying *yasirān*, because the motion[3] will then be
weak, and that/ is because the extreme shortness 5
of the shadow is at the extreme elevation of the
sun, and elevation is the position of/ spiritual 6
people and dominion, and unto it are raised the
hands of the suppliants and (upon it) are fixed
the eyes of/ the fearful. And the sky, although 7
all of it is elevated, the zenith of each
inhabited locality/ is the highest for it. So 8
the meridian circle is the extreme of altitude
for moving things in it (the sky).

It is said, with regard to "grasping" 9
(*al-qubḍ*), that it is annihilation, because the
conclusion of things and their destiny is/ God's. 10
There is no use commenting on him who said,
"Verily the extension of the shadow is between
the dawn until/ sunrise". It means its being; 11
so it would have been necessary to say, "Verily

[1]Text في سكون ; MS مافي سكون.
[2]Qur'ān, 25:46.
[3]Text الحركة ; MS تحركه.

it is from the rise of the dawn until/ the s49:12
setting of the twilight", and because of the
continual roundness of the circumference of
this shadow that is apparent to the eyes/ in 13
lunar eclipses in [different][1] positions of
the heaven as to longitude and latitude./ By 14
measuring[2] it the size of mountains on the
earth can be determined, as Manṣūr b. Ṭalḥa
found out at the vanishing of the moon./ But 15
there is no protuberance or hollow on its body
in the shadow, but then he said they do not
show, because of the smallness/ beside (the 16
size of) the moon, like the smallness of the
mountains beside (the size of) the earth. So
they are necessarily hidden just as/ the trace 17
of the mountains is hidden in the circular
shadow of the earth or by the grossness of what
is perceived. So then for it/ the shadows are 18
perceived, but the position of the moon with
respect to the sun differs, and it entails a
difference/ in their shadows as to the size of 19
their positions in the course of the month, but
the disappearance as its condition and shape
is not/ variable, and hence there are no s50:1
protuberances or hollows.

It is said that there is a body other 2
than the earth, of opaque construction, with no
transparency/ in it. It accepts the light in 3
the way the earth accepts it (the light), it
being the moon, [capable][3] of erasing (the light)./
Its pine-cone (shaped) shadow extends like its 4
(the earth's) shadow and its point (*sahm*) is along
the prolongation of the line joining/ the center 5
of the sun and its center. These two shadows
differ in amount/ because of the two objects whose 6
shadows they cast, the body of the moon being

[1]Text مختلة ; read مختلفة .
[2]Text برازه ; MS بزاره .
[3]Text دون ; MS ذو .

approximately a part in forty parts of the body s50:6
of/ the earth, and the body of the s a hundred 7
and sixty-six times it (the earth). And they
differ in distance, for/ the distance of the 8
moon from the earth is a part in nineteen parts
of the distance of the sun/ from it. They differ 9
also in position, for the shadow of the earth is
always extended between the earth/ and the 10
heaven the direction the sun is away from it,
but the lunar shadow, because of the difference
in distance between/ the two luminaries during 11
the course of a month, is other than constant
in situation, for sometimes it is toward the
earth/ and another (time) in a contrary direc- 12
tion to it, up. That, at conjunction and
opposition/ is between the two, but is not 13
perceived by the eye except at solar eclipses.
Then it is determined/ by measurement, when the 14
moon's light is different, increasing from (its
time of) waning until full moon, and decreasing
from/ then until the last night of the lunar 15
month. And because this light falling from the
sun upon its (the moon's) body is reflected
back/ to the earth and illuminates from its face 16
what(ever) is opposite it, there occurs for the
earth also from its side a/ pine-cone (shaped) 17
shadow different in position from its shadow
caused by the sun, I mean that the vertex of the
cone/ for it is in the direction of the moon, 18
and it, from the direction of the base, its
extension is increased until the/ sun's ray 19
overwhelms it and its trace is reduced to nothing
by it. As for the planets and the fixed stars,/
we who investigate the truths of the existent s51:1
forms see them as self-luminous things/ like 2
the sun. Some deem them not self-luminous, but
gaining their light/ from the sun, like the moon. 3
Uncertainty as between the two opinions exists
among the peoples since it has not been decided/
between them by a necessary and// direct proof 4
depending on the laws of learning. f. 128b

Verily the difference is known then, s51:5
from what we inferred, between darkness and
shadow, and how/ the two kinds come from one 6
kind. We say that being illuminated is a quality
which is possessed by a/ non-transparent body 7
when it is confronted by a luminary, together
with a transparent (medium) being in between
them. So/ that transparent thing will permit 8
the passage of all of the light through it, but
it will be the portrayer (*lit.* result, *ḥāṣil*)
of the colors and shapes facing it./ And in fact 9
confrontation requires straightness of the
distance, and hence the ray(s) of/ the two 10
luminaries and the stars and fires are seen
in rectilinear extension until they are
necessarily/ concealed from the senses. When 11
the source of illumination disappears at the
head of its prolongation/ that acquired state 12
ceases, and it becomes dark. And [since][1]
darkness is the absence of light, and shadow
is/ the absence of illumination, hence the 13
opposition between the two is the opposition
between nullity and being, and not an opposition
between two existent,/ incompatible things. 14
This is the matter as to the situation which
obtains when visual perception (occurs),/
whether it is for the object seen, according 15
to the opinion of Galen concerning it, and
the geometers, or whether at/ the [eye][2], 16
according to the opinion of Aristotle, who sees
it with more validity than the first.
Verily the controversy over it lengthened 17
in the direction of mutual exasperation among
the leaders of the two opinions, along with/ the 18
advancement of the geometry of optics (*manāẓir*)
from each of the two schools of thought equally.
[Be that as it may][3]/ this rectilinearity in the 19

---

[1]Text من ; read متى .
[2]Text البصر ; MS المبصر .
[3]Text لديما ; read لكنها .

solar or visual ray is bent, together with its s51:19
penetration, like its bending/ at the common s52:1
part between two bodies differing in transparency,
because of the purity/ and density differential 2
in the elements of the two, an example being the
difference of air and water for thinness, and
fire for density./ This bending is called refraction, 3
like the (apparent) break in the case of a straight
(object), but it is not/ attributed exclusively to 4
[water with][1] air only; it is common to other
transparent substances, whether a watery/ fluid 5
or a limited solid, provided there occurred in
it differences in density/ and thinness, together 6
with the absence of (any) mixing. So each one of
them stopped at a [place][2] just as/ the standing 7
of water and oil (*duhn*) in one vessel by being
contiguous only (i.e. not mixing), and verily/
the common part between the two of them bends 8
this straightness so that there result from it/
marvelous things in water, and crystal, and 9
things like them.

As for (the effect of) smoothness and the 10
lack of penetration, this straightness bends/
with reflection as we remarked in connection with 11
its bending at the surface of water and the surfaces
of mirrors having different (kinds of)/ surfaces, 12
so that one perceives by them (something) different
than what is the object of looking and contrary
to (its) form, and there results/ from it also 13
marvels in the vistas of the air, and by it are
[constructed][3] burning instruments./ Air is not 14
affected by light when reflection occurs in it
preserving/ equality of angles. Rather, on the 15
contrary, it is not seen in the case of a concave
mirror in the shape of a cone with vertex/ at the 16
burning point if it is set up along a ray of the
sun falling in a/ wide[4] house. 17

---

[1]Text الايح ; MS الادمع .
[2]Text حيرة ; read حزّه .
[3]Text معد ; MS صنعه .
[4]Text واسع كرية ; MS واسع من كونه .

So if someone assumes that this cone is s52:18
among the shapes scattered in/ the air which do 19
not appear except in the ray(s) of the sun piercing
through holes into their houses(?),/ he will s53:1
realize that he is correct in his thinking to
visualize the matter in its essence, and that is
that the air,/ since it attains the extreme of 2
purity and its freedom from colors not perceived
by sight, for the eye/ indeed perceives colors 3
upon which light is falling, and in (the act of)
perceiving, one cannot dispense with/ a transparent 4
medium between him and them. Hence shapes and
what is connected with perceived (objects)/ as 5
to motions and differences in position are
perceived by means of colors. The distinguishing
between them of the sense common (to all)/ is 6
by the strength of measurement after training
and experience. So the lighting of the air is
not/ sensed by it, but the ray piercing through 7
holes is sensed by the place where it falls//
on/ shapes, they being the solid terrestrial f. 129a
parts, non-transparent, illuminated,/ and joined 9
by their multiplicity. So it will be seen in
such fashion as to [divert][1] from the perception
of what is behind it. So it is no wonder that/
the cone which is seen in the concave mirror is 10
among the kinds of shapes, but/ there is no 11
difference between it and the rest of them which
necessitate its being a cone other than what I
am saying./ That is, the shapes which are 12
receiving the ray are illuminated from above
only/ and are [in shadow][2] below, and their 13
shadows are almost sensed if the hand is put/
under the greatest of them near it. Most of them, 14
to sum up, are seen in one condition because of/
their smallness, even if there is a difference in 15
them. If the mirror in that ray is set up
opposite/ the sun's eye it is reflected from it, 16

---

[1]Text تشتعل ; read تشغل .
[2]Text مضلة ; MS مظلّه .

it being concave upward (? *sāfila*), to the s53:16
burning position, which is near/ its center, 17
above it at approximately half the distance
between them, and so there results from that/
reflected ray a cone extending up from below; 18
so it lights the lowermost of the shapes/ which 19
are in its path, which before that were dark.
So it differs from others by the doubled light/
and total illumination, and the distinguishing s54:1
of the cone of light, so that it becomes percep-
tible and sensed.

These shapes also (are) because of the 2
differences of shadows as to the quantity of
darkness,/ and that is because whether the 3
shadow is from a gnomon set up, or a built wall,
or/ from the ceiling, if its amount does not 4
become large because of the great increase in
distance of its ends, so then the air/ around 5
about it is lighted by the forms which are in
it and there reflects from each of them/ some- 6
thing of what falls on them of the rays
(reflected) to others than it, and the successive
reflections join/ at that which is in the air of 7
the shadow. So there results in it distant
(hence weak) illumination until that can be
described/ as very distant. So then the darkness 8
reasserts itself, and thus is the situation
inside houses./ The ray which enters it illumi- 9
nates of its wall what is opposite the sun's eye,
which is self-luminous./ Then it is reflected 10
from it to something else and it lights it up
indirectly and the illumination is weaker than
the first, and so on/ until it is reduced to 11
nothing. If the penetrating ray is traced into
the house while being looked at, and/ a person 12
other than he agitates clothing or something
white in the ray behind him, even if it is not/
smooth the observer will perceive that motion 13
upon the opposite wall/ by the increase of light 14
and its motion.

As for that which Aḥmad b. al-Ṭayyib 15
al-Sarakhsī mentioned in his book called/ "The 16

Elements of Philosophy" (*Arkān al-falsafa*) s54:16
concerning the blackening of the air at the
heights of lofty places, verily he/ exaggerated(?) 17
the opinion of Aristotle concerning the blackness
of the air according to what appears of his words
in/ the book "De Sensu" (*Kitāb al-ḥiss w'al-maḥsūs*) 18
which relies on experience and example/ by trial, 19
without (relying on) information (from others),
and he does not transmit to us information about
this blackness and the absence of the sunrise/
[from][1] the backs of lofty mountains. They do s55:1
not mention variation in it like what he mentions
with regard to the intensifying of cold/ or the 2
absence of heat. Since Mount [Demavend][2] is so
high we have indeed witnessed (it)/ and others 3
than we have witnessed (it) at [the top of][3] its
summit, and the back of its peak. Then he mentions
nothing of that blackness/ even if it does not 4
occur (?). Undoubtedly the Caucasus Mountains
attain extreme loftiness (and we have)/ Aristotle's 5
acknowledgment of it in the book "Meteorologica"
(*Kitāb al-āthār al-ʿulwiya*). He adduces reasons
for their height and claims that/ vapor does 6
not ascend to them and that winds do not reach
as far as them. He infers this from the permanence
of lines and marks/ made in the ashes of sacrifices 7
and (animals) sacrificed on them (remaining) in
their (original) condition without/ being destroyed 8
by wind or being effaced by rain. He mentions
nothing in it about the blackness of the air.
For if it (the air) had been (there)/ he would 9
not have known the customs and the deeds which
were performed on them during/ their early 10
ignorance (i.e. before Islam). They say that
that darkness is more marvelous than other things;
they even/ manufactured fables about it to 11
strengthen the beliefs of those ascending them

---

[1]Text ممن ; read عن .
[2]Text دیناوند ; MS دنباوند .
[3]Text من ; MS من علاء .

with the sacrifices,/ and those listening to s55:12
them// at the time of (their) return. f. 129b
We see that the air is (vari)colored, 13
and not everything without a color is described
as/ black, it being one of the colors, not the 14
absence of them, and the existence of the sun
opposite these/ summits necessarily implies their 15
illumination, like the illumination of the
mountain sides, and the low parts, even if they
are not reached/ by vapors or shapes, as the peak 16
of the mountain which Aristotle described is
illuminated/ in the direction of the summer 17
solstice from the eastern direction before sun-
rise on/ the earth, by an extended (length of) 18
time.

There results from the saying of Aḥmad 19
that the heavenly bodies are not luminous, and
that/ the cause of their light is from below, s56:1
and is not present except to an observer of it
(looking) at them (from below). So it is claimed/.
The difference between the sun and the moon is 2
asked about, and the situation differs from one
of them to the other, [not] both of them being/
self-luminous. The misfortune for these people 3
is from their exaggeration in taking sides with
the opinions of Aristotle/ entirely, and in 4
their belief, excluding the possibility of
error in it, in spite of their knowledge that
he was one of the deep thinkers,/ but not one 5
of those who are infallible. Deep thinking, even
if it is exaggerated as a cure from the danger/
of errors, this being the cause of their fathers' 6
complaint and of the suffering of their nature
and manners;/ they permit themselves to obey the 7
entirety of the "Meteorologica" of/ Aristotle, 8
for what is there in it about the ray of eyesight,
as though it is not contrary to his opinion/
except as to the words. [They attribute it][1] 9

[1]Text يصلفونه ; read ينسبونه (?). The preceding sentence is incomplete. Perhaps the text is garbled.

to others than him in order to make him noble, s56: 9
and if one of/ those studying cosmography as it 10
really is denies some of the abominable errors
in that book,/ like the lack of inhabitants 11
(of the globe) under the summer solsticial
tropic (the tropic of Cancer) and their complete
absence behind it/ in the direction of south 12
(the partisans of Aristotle) agree[1] on denying
the evidence by refuting it, and so they became
ridiculous/ with their trying to purify his name 13
from error.

Indeed, I composed a treatise devoted 14
wholly to that and I called it "A Disclosing
of the Burning Method" (?) (*Al-Ibāna ʿan al-*
ṭarīqat al-muḥtaraqa(?) ./ So they do not only 15
confine themselves to these opinions, but they
consider them as standards. (This) prejudging
of/ their results resembles (alleged) eyewitness 16
stories, like the blackness which al-Sarakhsī
tells about concerning/ the air at the summits 17
of mountains, and like their measuring the
quickness of freezing of hot water, because of
its mildness/ and the looseness of its parts, 18
before the freezing of cold water, because it
is dense and its parts compact. 19

I put in each of two equal and similar
vessels/ equal amounts of pure water, cold and s57:1
hot, (at temperatures) which do not feel painful
to touch,/ and I exposed them in clear air at 2
the same time. So the cold surface froze, but/
some heat remained in the hot one. I repeated 3
that once again, and I elevated the hot one
very much/ and the cold one froze. But the 4
hot one did not reach the degree of heat of
the first one. After that is their saying
regarding/ the air of an underground conduit, 5
that in the winter its heat is in excess of
what it is in the summer, and conversely./ But 6

[1]Text تالبوا ; MS تانبوا .

the experiment as to the time of solidification s57:6
of wax or melted suet, for each of these/ two 7
seasons, and the preserving of an amount of
hair from [clothing][1] which drives away harm
only in/ them (the seasons) refutes them and 8
corrects (the notion) that heat and cold are
two qualities/ connected with air. 9

That which is next to the surface of 10
the earth is conditioned by them (heat and
cold) more than the conditioning/ of the parts 11
which are farther from it. I am accustomed
to one of the learned (men) of the partisans
of Aristotle/ telling that if this is correct 12
it does not [controvert][2] what we have of the
natural sciences. So I said to him, "Verily
it will/ [controvert][3] the elements upon which 13
you built. And if they are [controverted][4]
and not valid,/ the science (built upon them) 14
is not called a science".

As to the laws of natural conditions, 15
they do actually/ exist. So if true knowledge 16
of them is attained these laws are then called/
natural sciences. But is not human knowledge, 17
together with what the parts make it, to be
reckoned by its amount in/ absolute investiga- 18
tions? Rather it is like the mountains and
the observers' conjecture, and we ask God/
for increase of goodness; verily He is the 19
guardian of goodness.

---

[1]Text الثباب ; MS الثياب.
[2]Text ينقص ; read ينقض.
[3]Text تنقص ; read تنقض as in the MS.
[4]Text انتقصت ; read انتقضت as in the MS.

THE THIRD CHAPTER s58:1

ON THE VARIATIONS TO WHICH

SHADOWS ARE SUBJECT IN AMOUNT 2

Verily, that which is connected with shadow 3
as to variations is of two kinds. One of the two
has to do with/ difference in position of the 4
source of light (along a direction) parallel to//
the dimension (*quṭr*) which bounds the height f. 130a
and lowness, it being/ the dimension of thick- 5
ness and depth. This variation is expressed
by altitude. The second has to do with/
difference in position of the source of light 6
(along a direction) parallel to the (plane of
the) other two dimensions, I mean/ length and 7
width, and it is expressed by direction (*or*
azimuth). As for the first kind, it affects
the shadow by/ increase in its extent or with 8
a decrease by contraction.

As for the second kind, it is connected 9
with a difference in position, together with
equality (i.e. no change?) in/ size. Both 10
situations exist simultaneously among celestial
sources of light./ So altitude does not vary 11
except with variation in azimuth, and their
situations are portrayed by isolating (them)
in/ the imagination. So the difference in 12
altitude is made one (apart from) the azimuth,
and the difference in azimuth one/ from the 13
altitude, because these two situations, even
if they are found at two different times, the
imagination/ does not cease picturing them as 14
following (each other) and in a motion which

is not (actually) present in the sky./ I mean s58:15
the motion of the elevated along one of the
circles of altitude so that/ the azimuth 16
remains fixed in its situation together with
difference in the altitude, or along a single
almucantar to keep the/ altitude fixed at its 17
amount. So that is not among the things which
are/ incapable of being pictured among the 18
first principles, like the impossibility of
two bodies being in one and the same place
together,/ or the presence of two opposites in 19
one place together and at one time. Verily
these fail to exist/ only because of the s59:1
contradiction in their existence itself, like
the earth, (which) the imagination does not
picture as/ touching the atmosphere, but out- 2
side it, nor whiteness in the feathers of the
crow. So the imagination/ does not recoil 3
from picturing it as white, together with the
cessation of the blackness from it, otherwise
actuality would be contrary to/ this portrayal. 4
For altitude has an extreme at which the shadow
itself disappears, and the other (extreme),
being/ its beginning; at it the extremity of 5
the shadow disappears. It is like a single
distance: if it is measured from below/ it is 6
called thickness, but if it is measured from
above it is called depth. Thus[1] it is/ for 7
the altitude: if it is measured from its
beginning it is called an altitude, but if it
is measured from its end/ it is called depression. 8
Otherwise the name of depression in the profession
is bestowed upon the opposite of the altitude,/
under the earth. 9

Hence in the naming of that I limited myself 10
to the complement of the altitude, and if the
altitude does not reach,/ along the luminary's 11
day-circle (*madār*), up to that extreme (of the
day-circle), wherever it does end is an amount/

[1]Text كذلك يجوزان كان ; MS كذلك يجوز كأن .

corresponding to the shortest shadow, that (in s59:12
the case where the absolute maximum is attained)
being where its arc of daylight is halved, with
(the great circle through)/ the pole and the 13
zenith, prolonged. Hence the shortest shadow
for the day is called the/ noon (*or* meridian) 14
shadow, and its direction (*or* azimuth) is along
the meridian line bounded by two points, the
north (point)/ and the south. The equinoctial 15
(*or* east-west) line, which is bounded by the east
and west (points), intersects it/ in right angles. 16
So there results from these two lines, front and
back, right/ and left, by comparison with the 17
animal (sic). But (putting) the matter thus is
not necessary, and it does not impugn/ what was 18
set by Aristotle, the east as the right of the
heaven, in spite of the agreement among different
peoples/ to call south in their language right 19
(*yamīn*) and the opposite to it left (*or* north,
shamāl). As a result height/ and lowness halve s60:1
the diameter passing through the zenith and the
nadir, and/ the noon shadow falls along its line, 2
and the shadow falling along the east-west line
along the right/ and left [bounds?][1], and the 3
front and back. Hence what is in between them
is measured as azimuths from them./ Verily, 4
between the right and the front is a quadrant of
the horizon circle. If the source of light
vanishes (i.e. sets?) from the direction of/
these two lines, the amount of its inclination 5
is measured from one of the two, it being called
the distance of the azimuth,/ or for simplicity 6
azimuth, and it is added to it. Sometimes it is
added to the east-west line/ and sometimes the 7
meridian line. The azimuth of the shadow is
always opposite (in direction) to the/ source of 8
light. Hence their amounts coincide, but the
sides of the line from which the measurement is
taken differ,/ together with difference in 9

[1]Text حدق ; read حدّى ?

direction from the other line. As for the east- s60:9
west line, it is/ thus named because the shadow 10
of// the gnomon is along it at sunrise on one f. 130b
of the/ two points of intersection on which 11
the night and the day are equal.

(Some) people call it the equating (*or* 12
equatorial, *istiwā'*) line because of the
equality of the day and the night/ whenever 13
the equinox occurs. But according to the
people of the craft, the *istiwā'* line is/ the 14
name bestowed upon the common part between the
plane of the celestial equator and the surface
of/ the spherical earth, it being the line of 15
zero latitude. So because of that its use was
disapproved of in/ this context, so as not to 16
confuse the nomenclature by the similarity of
the names. It is also called the east-/west 17
line (*khaṭṭ al-mashriq w'al-maghrib*), because
it ends at their hearts, and is the mean between
the kinds of each sort of the two of them./ The 18
meridian line is called the line of declining
(*khaṭṭ al-zawāl*) because (upon arrival) at it
the sun declines from/ the meridian circle. 19

It is mentioned in the books of the ancients s61:1
as the noon line, and the declining (*zawāl*) is
an expression of the religious law by which,/
(when it arrives) prayer is made lawful, but 2
prohibited for a space of time before it, it
being the presence of the sun/ in the meridian 3
circle. But for the true instants of time, if
(the sun) actually(?) were at it (the meridian),
and verily/ the perfect (time) of the deed is 4
not just at it, but it is connected with a time
other than it. So the time/ of prohibiting 5
the prayer is the time at which the sun, according
to the senses, has stopped. Thus/ it is said that 6
then the sun abstains (*or* fasts, *ṣāmat*) (just)
as it is said that the wind is abstaining during
its stillness,/ and the horse abstains during 7
its refusal to partake of fodder.

The poet said, s61:8

> It (the sun hesitated a little 9
> and then spread
> A minute shadow among the old
> and thin [Spanish reed][1]
> trees.

Dhū al-Rumma said, 10

> And the sun hesitates, lingering 11
> about in the heavens.

And he (also) said, 12

> At its head the long-stationary 13
> sun.

Among the people are those who added to that 14
and made of it then a rotation on/ itself, like 15
a thing which is restrained from its forward
motion, so it is curved away and there arises
a rotation from/ its being curved away, if it 16
does not turn backwards. So if that (meridian
transit) is measured by the altitude of the
sun/ or the amount of the shadow it would become, 17
for this time, an appreciable latitude (of
error), because the variation in the/ solar 18
altitude at it will not subsist except by
what minutely impinges on the senses from the
parts, and likewise for the shadow (also)./
However, if it is measured by the azimuth of 19
the shadow, and the instrument is made very
large, the latitude (of error) at the (above-)
mentioned time will become/ less. The difference s62:1
of azimuth at that time even though it also is
very small, it is seen at/ different altitudes, 2
and it is called the noon shadow (*ẓill niṣf al-
nahār*) also. Verily, [concerning the *zawāl*][2],
the Arabs, as we said,/ call shadows from sun- 3
rise until its setting, *aẓlāl*. So they

---

[1]Text ا لاباصيل ; read ضئيل (Arundo Donax =) الإباء .
[2]Text فان العرب ; MS فان في الزوال العرب .

particularize by this/ name what precedes noon, s62:4
and what else there is after it (they call)
afyā' (pl. of *fay'*), just like calling what is
before noon during/ the day morning, and what 5
is after it evening. The reason for the nomen-
clature there is that *al-afyā'* is from the
inclining/ and the return. As for the 6
inclining, the shadows incline from the side
of the west to/ the side of the east. 7

As for the returning, they indeed return 8
to their first magnitudes. As for the shadow,/
if it covers (the places) well hidden from the 9
lights, whether from the sun, or the moon, or
a fire. The shadow from/ the moon is specially 10
named separately, it being *al-samar*, and by
another *al-fakht*. It is said that/ it is the 11
color of a ring-dove (*al-fākhita*), like what
is said of *al-samar* that it is a dusky (*asmar*)
color, and that it is so called from the fact/
that boys of the quarter were conversing 12
(*yatasāmarūna*) at night about it, but I have
never heard anything on it about using *al-fay'*/
to the effect that it is said concerning 13
al-fakht that it is in the first (part) of the
night, but as for its end it is its nickname./
Some of them assert the contrary as to *al-fakht*, 14
making it moonlight.

Among them are those who use it (*al-fakht*) 15
as being both for its shadow and its light,
however, revelation/ has settled (the matter) 16
between the two sides as to the likenesses of
shadows. God, be He exalted!, said, "Do they
not see/ what God has created of things shading 17
them in their shadows from the left and the
right, prostrating themselves to God, they
being/ humbled." "And unto God those who are 18
in the heavens and the earth prostrate them-
selves in obedience and in spite of themselves,/
and their shadows in the morning and the late 19
afternoon".[1] Measurement necessitates that the

[1]Qur'ān, 16:48 and 13:15.

noon shadow not be called/ *fay'* because it is s63:1
a stopping (time) between the increase and the
decrease and it is not counted as being from
one side/ to the other, but there is no 2
[confusion?][1] as to what has come to be under-
stood by the nicknames. However, as to the/
prostration of the shadows, prostration 3
originally being the nodding of the head and
inclining, so that/ the inclining honeybee is 4
described as// f. 195a

Here the displaced passage
from Ibn Sinān ends, and the
printed text resumes.

prostrating itself, which implies 5:10
indication, like His (i.e. God's)/ saying, be 11
He exalted!, "There is not a thing but
celebrates His praise."[2] The recitation of
praise in the essence of a thing/ is its rising 12
to its perfection, and its seeking peace[3] by
nature with the divine object in its continuous/
existence for what it was created to perform. 13
What transcends this about it is its indication
by the various (forms)/ which it undergoes and 14
the (different) appearances which it takes, (in
which case) it is restricted, derived, and to
be explained.

One who seeks guidance from Him about it 15
becomes a praiser like it, and he does not have
to [perceive][4] (directly)./ For it is like His 16
saying, be He exalted!, "The stars (*or* herbs)
and the trees prostrate themselves!"[5] That is,
the two of them possess shadows[6]/ which are cast 17
as in prostration. It is as though the kneeling

[1]Text مشاحة ; read مشاجة .
[2]Qur'ān, 17:44.
[3]Text رؤابه ; MS دروبه .
[4]Text has يقفه ; read the MS ىفقه as يفقه .
[5]Qur'ān, 55:6.
[6]Text ذواظل ; read ذوظل

is from self-abasement and destitution, and 5:17
every/ creature is obligated to self-preservation 18
and thrift.[1]

It is said also, with regard to the prostra- 6:1
tion of these two, (that it is) the obedience
for growth to the given extent/ for the preserva- 2
tion of nature, and that goes back to (something)
which is not far from what we mentioned about it.

Indeed it is said concerning the stars 3
(mentioned in the quotation above) that the
planets (? *al-kawākib*) are intended, and that is
not impossible, since/ obtaining (astrological) 4
indications from the planets by their motions is
without a medium, whereas from plants/ (there 5
is a) medium. So nothing keeps accompanying
things like their shadows, whether the sun/
indicates their bounds or not. Thus the shadow 6
of a gnomon extends horizontally on the ground
like/ the kneeler placing his head on the ground, 7
throwing dust on his face, with his shadow moving
from one side to another,/ being carried (by the 8
sun) from one place to another, and from one
side to another, indicating its cause,/ which is 9
the motion of the sun from sunrise to sunset.
It is among the most mighty of indicators/ and 10
the most clear (indicator) of them of the Prime
Mover who moves.

The shadow, which is the closest of things 11
to the human, (is) the distant ladder/ of inference. 12
Thus it is his prostration, whether its owner is
mindful of it concerning/ the duty and is 13
[performing it willingly][2], or not paying need to
it and not performing his duty. Parts of him are
kneeling/ while [some][3] not. Others are obtaining 14
indications from it while others are not. So the
mind imposes upon/ its possessor: (a) the obtain- 15
ing of indications from [the like of it][4] (the

---

[1]Text has العبقبة ; read التنقية as in the MS.
[2]Text طالعا ; MS طابعا .
[3]Text لعبض ; MS بعض .
[4]Text بمثاله ; MS بمثاله ; read بمثاله .

shadow) which is moving while he himself is not, 6:15
without leaving him/ or separating from him, 16
and (b) the consideration of his various forms
by its (the shadow's) variation, and (c) not
leaving/ a volatile thing called the playground 17
of his shadow, by virtue of which he dispenses
with other things, and (d) not be/ like the 18
ignorant (who is) afraid of his shadow, but on
the contrary to know that it is impossible to
stop/ the shadow from prostrating itself or to 19
transport itself from right to left.

Verily God, be He exalted!, mentions only 7:1
the early morning and the evening because of the
excessiveness of extent of/ the shadow at the 2
two (times), and its close resemblance to
prostration at those two (times), with the
[shadow-caster][1] being erect, since/ it is 3
possible[2] for the [shadow][3] to be shrinking
instead of stretching by changing the position/
of the shadow-caster, and tilting it away from 4
perpendicularity, as Abū al-Fara[j][4] b. Hind
said,/

> Unto us is a king to whom none of 5
> the functions of royalty
> appertain.
> Except that on the day of peace he
> puts on the crown.
> He is supposed to reform people 6
> while he himself is corrupt.
> How can the shadow be straight when
> the rod (casting it) is crooked?

This simile was taken in the two (above) 7
couplets from the saying of Ibn Thawāba/ when he 8
was asked about Ṣāʿid, and so he said,

---

[1]Text الظل ; MS المظل .
[2]Text الممكن ; MS من المكن .
[3]Text للمظل ; read للظل .
[4]Text فرج ; MS مرج .

> "He is the one the shadow of whose cloak[1] does not exceed his person." 7:8

Of the saying(s) of Abū al-Fatḥ al-Bustī (is), 9

> You have become a mule by depending on the current proverb: 10
> If the dagger is curved, crooked also is its sheath.

Also that its motion at these two times will 11 be most apparent,/and that there is need of 12 motion for it in order to indicate the motion of the source, and one should be mindful/ also of 13 the great differences in the shadow's motion in spite of the regularity of the solar motion/ both 14 to observation and investigation. And also that the shadow at these two times is indeed extended,/ its head [stretched out][2]. Thus its companion 15 is the slave, who is not the master of his (own) head.

One of the reasons the Christians (adduce) 16 for facing the east (in prayer) is what is in the Gospel/ to the effect that Mary Magdalene in the 17 morning went to the tomb of Christ and saw on/ the road a shadow preceding her. So she turned 18 and behold it was the Christ. Verily his shadow prostrated (itself) in this/ story. So to whom 8:1 would it prostrate itself? [Would that you knew][3] if it were a deity? But in fact, the shadow turned away,/ prostrating itself to someone else, 2 witnessing that the shadow-caster has a master. And since Mary saw the Christ when/ she turned, 3 he facing the west, from which (direction) the Christians turn, according to the rules/ of their 4 sect, which is// [contradictory][4]. Moreover, f. 195b

---

[1]Text وزارته ; MS وزراته .
[2]Text مدوم ; read MS as ممدود .
[3]Text ليت شعري ; read ليت شعرك as in the MS.
[4]Text ناقصوا ; MS ناقضوا .

these two times, sunrise and sunset, are the/ 8:5 worthiest of times for the knowledge of motion of the rising and the setting (thing) as to the variation of its shape in/ appearance, as 6 Abraham, upon him peace!, deduced about that at these two (times).

To this is referred in what is reported 7 concerning Abū al-Dardā' that he said: "If you want to,/ I am ready to swear that the 8 nearest of the worshippers of God are those who observe the sun and the moon/ and the stars and 9 the shadows for mentioning God (i.e. worship)". He means the shadow, since it (induces) the virtue of meditation about the creation of/ the 10 heavens and the earth, and its use in affirming the (Muslim) creed and at the times of devotion.

However, as to what was reported to the 11 effect that the ruler is the shadow of God on His earth, its meaning is directed/ toward one 12 who is learned, not toward one who rules by conquest. For how could it be directed toward him, together with what is said,/ "(One who 13 is) created should not be obeyed in matters contrary to the will of the Creator". By this report is meant only (that) he who accepts/ His acts, be He exalted!, as to keeping the 14 people under the rule of equality, and showing them the ways of benefit,/ so that a person's 15 shadow resembles his deeds which move with his motion and stays with his staying,/ unless he 16 errs, due to what is in his own (animate) clay. As Abū Bakr al-Ṣiddīq said regarding his animal instincts,/ "Unto me is a devil (who) 17 possesses me, and whenever he makes me crooked, straighten me out". However, as for one who [spoils][1]/ the earth intentionally and wilfully 18 destroys the land and harmfully counters the acts of God, God raises/ Himself above having 19 as His shadow the like of him, or of (having) him as the judge of His creatures.

---

[1]Text يبعث ; MS بعيث ; read يعيث .

THE FOURTH CHAPTER 9:1

ON WHAT THE EXTREMITIES OF THE 2

SHADOW DRAW ON HORIZON (PLANES)

The first (part) of astronomy is based upon 3
the insignificance in the size of the earth of
the senses in comparison with/ the ecliptic, (and 4
the fact that) the plane passing through the convex
surface of the earth tangent/ to it at (the 5
observer's) locality represents the horizon plane
bisecting the (celestial) sphere and/ replaces 6
it. So the end of the gnomon, hence, is like the
effective center of everything. The sun draws,
in/ a day and a night, by the total motion, an 7
apparent circle, as a first approximation. For/
its motion is (in fact) along a spiral line in 8
shape, joined. The rays emitted from/ that circle 9
to the head of the gnomon which is always the
center draw a/ cone of rays whose vertex is the 10
end of the gnomon and whose base is the daily
circle of the sun.

That straight ray describes in the heavens 11
a circle equal to the/ sun's daily circle, coin- 12
ciding with it as to the amount of its declination
(but) in opposite direction. It (also) describes
a cone,/ like the first, called the shadow cone, 13
because the ray, when it passes through the head
of the gnomon,/ borders the shadow which results 14
from it. And the horizon plane necessarily cuts/
the shadow cone. If (one is) at the (terrestrial) 15
equator, it (the horizon) will be parallel to
the axis of these two cones./ Hence the ends of 16
the shadows will describe straight lines. If
(one is at a place) other than/ the equator, the 17
plane of the horizon declines from parallelism

with the axis of the shadow cone and meets/ the 9:18
axis inside the cone of the rays. So the end of
the shadow describes on it/ curved lines known 19
as hyperbolas which continue until the latitude
increases (sufficiently), whereupon they become,/
a parabola upon the horizon plane's (attaining) 10:1
parallelism with the side of the shadow cone
which is under it, (but)/ not that which is above 2
it.

Indeed it yields a parabola because the 3
apparent horizon plane/ is not really the plane 4
of a great circle. Then upon the passage of
that position to/ one whose latitude exceeds the 5
complement of the inclination of the ecliptic,
some of/ the northern daily solar paths will 6
appear wholly and will be above the horizon, and
hence when the sun rotates in them/ the inter- 7
section of the horizon plane with the axis of
the cone occurs inside it, ranging from one side
of it/ to the side opposite it. Then the section 8
will be that called the ellipse, and because it
is a/ closed path, the end of the shadow goes 9
around the gnomon from all directions, drawing/
an ellipse, circular but elongated, so long as 10
the sun is in these/ daily paths. 11

Abū al-Ḥasan Thābit b. Qurra has a complete 12
and well-done book, "On the Determination of
the Lines Drawn/ by Shadow-Ends on Horizons" 13
(*Fī taḥdīd al-khuṭūṭ allatī tursimuhā aṭrāf al-aẓlāl fī āfāq al-arḍ*)./ Also Ibrāhīm b. Sinān 14
has discussed it fully in the "Book of Shadows"
(*Kitāb al-aẓlāl*). Because we have the extreme
of// the quadrant of a circle, then in a horizon f.196a
of (those) under the pole, the end of the shadow
describes loops which are/ in fact closed spiral 16
lines like the closed paths which/ the sun 17
describes.

Abū al-Ḥasan Thābit b. Qurra, in his interest- 18
ing problems, errs/ in his saying that the light 19
entering through a small hole into houses will
be cylindrical,/ and hence it will be cut by the 11:1
wall in an ellipse, as though the cylinder alone

pertains to this/ section, and not the cone. 11:2
For the (above-)mentioned ray is not cylindrical/
in shape, but rather conical. 3

Let the sun be *AB* (in Figure 1) and the wall 4
MS, and the hole *GD*./ Then the light entering 5
from it will not be cylindrical because the hole
is smaller than the sun./ But even supposing it 6
were equal to it, the entrance of the ray into
it will not be in the form of a cylinder/ *AGK* 7
(and) *LDB*, but there extends from *B* to *G* ray/ *BGM* 8
and from *A* ray *ADS*. Then the wall/ *MS* is struck 9
by the cone *TMS* in an ellipse. And however much
the hole is farther/ from the wall the section 10
will be larger, because the vertex of the cone,
T, is in the direction/ of the sun, and the 11
situation will be similar if the hole is less
than the sun,/ the ray being always conical; 12
it cannot be otherwise.

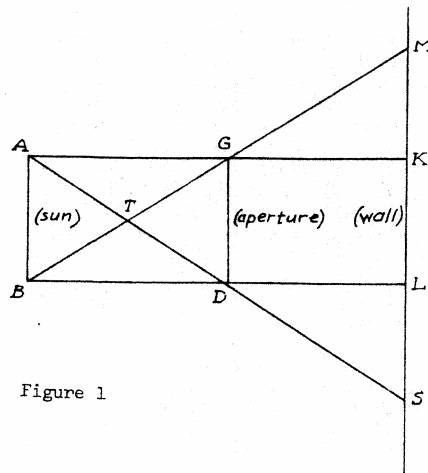



Figure 1

THE FIFTH (CHAPTER) 12:1

ON THE VARIATIONS TO WHICH A SHADOW IS
SUBJECT BECAUSE/ OF DIFFERENCE OF 2
SITUATION OF THE LUMINOUS OBJECT AS TO HEIGHT

If an intervening object (which is) non- 3
transparent casting a shadow on another like it,
with a source of light higher than both/ is near 4
to the other, its shadow will be a true (one)
because few/ reflected rays are reaching it by 5
dust (particles), or (because) they are weak, or
missing (completely) due to (the fact)/ that the 6
paths are obstructed. Also the edge of the
shadow is purer in shape and with [illumination][1]
less/ mixed, and of more apparent edge. But if 7
the distance of the shading object from the
shadow increases, the shadow/ and the illumi- 8
nated part will [start][2] to mix, their common
portion being ill-defined so that it cannot be/ 9
a pure shadow nor fully illuminated.

To this al-Kindī referred in his remark 10
[that][3] for any pierced surface, the/ closer it 11
is to the earth, the truer the shadow and the
brighter the illumination from the hole, and if
it is raised,/ the illuminated (part) widens 12
and the shadow becomes dustier and is mixed, as
we related, until it vanishes completely./

---

[1]Text الضاء ; read الضياء as in the MS.
[2]Text احد ; read اخذ .
[3]Text اي ; read إنّ .

Verily the shading curtain made of old rushes 12:13
will not shield completely due to the numerous
holes (in it)./ Its shadow becomes blended like 14
smoke, contrary to the shadow of a dense mountain,
with which is compared/ the intense blackness of 15
a ewe, thus they say (it is) black like the
shadow of a stone. And because the sun is
greater than/ the earth and any other shadow- 16
caster of which we know, the earth's shadow by
necessity/ becomes slender and sharper with 17
increase of distance from it (the earth). This
is the situation in the case of all other/
objects on it, so that if the two rays bounding 18
any two/ opposite sides of them meet, the shadow 19
will vanish entirely before meeting the horizontal
plane,/ and the ray passing from the sun to the 13:1
head of the gnomon (will vanish). For this
reason the/ [serrations][1] will disappear from 2
the earth's shadow, called the heaven of the
lunar nodes, even though/ the common portion of 3
the illuminated part of the earth's surface and
the other/ shaded (part) may be serrated by 4
valleys and mountains. For the same (reason) as
the/ earth's shadow, which covers the moon in its 5
paths, is mixed, the colors of its eclipses will
change (accordingly)/ due to the difference in 6
the intensity of the darkness and its weakness,
culminating in the intense blackness at the axis
of the/ shadow cone. 7

These are precisely the reasons for the 8
circular illumination from a hole having angles,
when/ it falls on a wall at a distance from it, 9
and that is due to the approach of those (edges
of the shadow) of any two successive sides/ from 10
the position of mixed illumination and light,
and their superposition/ to form a figure whose 11
distance from the corner exceeds its mixed (part)/
from the middle of the side, there resulting from 12

[1]Text التضرليس ; read التضريس as in the MS.

it a polygon having sides whose number is the 13:12
double of the angles of/ the hole. Then the 13
polygon also undergoes what happened to the
hole.// f. 196b

In this manner the number of the angles 14
continues to increase by doubling,/ like the 15
doubling at chess, by the property of the double
of the double. And as the multiplicity increases
greatly/ it is perceived as round. Verily, the 16
circumference of the circle, according to some
of the natural philosophers, is everywhere dense
with angles,/ and thus it is related both to 17
arcs and to angles.

One who seeks harmony among similar (things) 18
and discord among unlike (things) should stand
in a place/ part of which is in shadow and part 19
illuminated, and let *ABGD* (Figure 2) be in contact
with ray/ *B[E]ZG*[1] at *BG*. Then let there be 14:1
erected from the earth a gnomon into the air,/

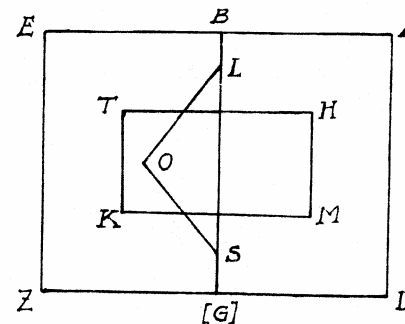



Figure 2

[1]Text: *BDZG*. In the text's Figure 2, *G* is missing.

and it will be distant from it so that it casts 14:2
its shadow, say *HTKM*. If all of the place/ is 3
illuminated, then what falls of its shadow in
the vicinity of the ray will [be found][1] in the
shape *LOS*,/ with the shadow reinforcing it so 4
that *LS* will be wider, but with the ray weakening
it so that/ its width is lessened and *O* is inter- 5
dicted from reaching *TK*, which would (otherwise)
be its position.

Perhaps the two rays may be combined, and 6
there results from them what is to be wondered
at by one who/ does not know the causes. For if 7
the sun's rays are entering a house from two/ 8
nearby holes in a screen or something else, and
if the two (resulting) circles on the floor such
as the two circles/ *ABG* (and) *ADG* (Figure 3) 9
intersect at the two points *A* (and) *G*, then an
intervening shadow-casting object/ between the 10
two and the sun whose shadow falls as *HTKM* will

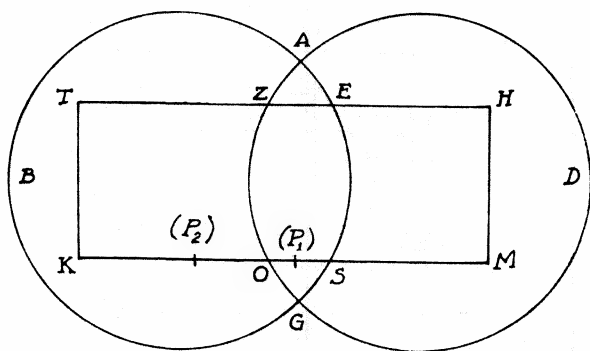



Figure 3

[1]Text سيجد ; MS سيجد .

have (a shadow) which divides itself into/ *ZKTO* 15:1
(and) *HESM* at the two ends (as) two pure shadows,
and the shadow will be annulled at/ *EZOS*. So there 2
will be illumination in the form of (the inter-
section with the floor of) the radii of the two
circles, because each/ of the two of them falls 3
in the shaded part with respect to the part common
to it and the other due to the difference/ in 4
situation of the two sources of light, there re-
sulting for the object two shadows, perhaps com-
bined finishing the/ darkness for the common 5
(part) but less so in what is not in common and
perhaps different (i.e. nonintersecting).

As Abū al-ᶜAbbās al-Īrānshahrī related in 6
(the book) "Problems of Natural Philosophy"
(*Masā'il al-ṭabīᶜa*), that he/ was on a shore by 7
the side of a mountain opposite the sun (where)
he found out the time, and that/ a man drew near 8
the shore. He was able to recognize on the moun-
tain two shadows (cast by the man), one of the two
above the other./ Then he (writes) at length 9
about the reasons for that.

So, let the mountain be *AB* (Figure 4),

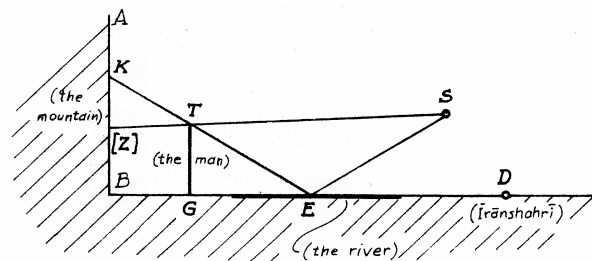



Figure 4

and the earth *BG* , and the river *GD*,/ and the sun *S*, and al-Īrānshahrī is at *D*, and the standing man at/ *GT*. And let there pass through his head ray *STZ*. His shadow on the mountain will be/ [*Z*]*B*[1], and let the reflected shadow from the surface of the water by two equal angles/ passing through the head of the standing one be *SETK*. So his shadow from it will be *KB*. But/ *ZB* is in shade because it falls in the shadow due to each of the two rays, while *KZ* will be mixed (light and dark)./ (This is) because it is shaded with respect to one of the two, while illuminated due to the other. A similar (phenomenon) is observed from/ a lamp with two wicks, or from two nearby lamps. (15:10; 16:1–6)

Let no one think that the above-mentioned polygons are ordered in/ time, but rather (they occur) simultaneously with sunrise. But reasoning (about them) requires (their being)/ ordered as (though they were) things formed one by one in time, and that is to (assist)/ the understanding. (7–10)

However, as to what we explained about the intersection of positions of mixed darkness and light, (it)/ can be verified by right (i.e. experimentally) if one stands with the sun on his right or his left and its altitude/ is about an eighth of a revolution (i.e. an octant), and// if he puts his hand on the parts of his face he will find the two shadows,/ of the finger and of the part protruding from the face, meeting and joining before/ the two parts meet. Indeed, I witnessed this in a similar situation, but I did not have the opportunity to/ examine it at other conditions of the altitude. (11–12; 17:1; f.197a; 2–4)

[1]Text *AB*. In the text's Figure 4 there are two *D*'s.

It is related of Plato that what he thought about it as to its cause is what we reported about the/ motion of the shadow toward another shadow, and that he said in the book Timaeus in his mention of/ matter (*al-hayūla*), that it is a shadow among shadows and that shadows flow from objects/ and are frozen by a highly spiritual device (which) condenses them to make shadows. (17:5–8)

Verily, it is said that flow occurs in all directions, then/ why do (we) not have shadow wherever there is light. (9–10)

We stated that heat keeps a liquid from [thickening][1], and cold, with/ a tendency opposite to that of light, [congeals?][2] it. The power of shadows to these people is obvious. Indeed they/ claim that if a hyena steps on the shadow of a dog walking on a roof it (the dog) falls/ off of it. And if a menstruating (woman) looks at her face in a mirror or touches it, it (the mirror) rusts. (11–14)

Because of the exaggeration of these (people) in the establishment of the effects of shadows,/ ʿAbdallāh b. Muḥammad al-Nāshī exaggerates in his turn (in the opposite direction), asserting that shadows have no effect (at all). (Indeed) he is confused as to/ what he was saying because of the excess of his resentment. He claims that the companions of truth (i.e. veracious scientists) denied what/ the astronomers gave as reasons for lunar eclipses, not accepting (the fact) that the sun's shadow will eclipse/ the moon because a shadow is not corporeal, in order (15–19)

[1]Text تكتنف; MS ىكىف, read تكثف.

[2]Text لجمه; MS ىحمعه, read يجمعه.

to do this, and that light [illuminates][1] darkness; 17:19
it does not remove/light. It was truly[2] said that 18:1
anger and haste are from Satan.

However, according to what was related about 2
Plato, a shadow should be a true (one)/ if it con- 3
tinues to stay, and that in winter it should be dense,
while in summer/ rarer, the silliness of which is 4
apparent since (the facts are) not thus.

The sayings of the man (Plato) are subject to 5
interpretation because of his use of special symbols,/ 6
for to him rest does not have a meaning opposite to
(that of) motion, except with regard to existence/ and 7
non-existence. Also the shaded and the illuminated
are to him opposite in this respect.

Then I say that if what we related about the 8
mixing of light and dark is determined, then/ the 9
amount of shadows will be (correspondingly) determined
as to their lack and deficiency (with respect to the
shadow) which should (have been cast). Because/ if a 10
gnomon is conical in shape, as used in instruments/ 11
made for (measuring) hours, then the ray at its tip
surrounds it on/ three sides so that it is near to 12
the height of the erected gnomon with the result that
its actual shadow/ is less than the shadow determined 13
(theoretically) for it.

What we mentioned can be observed if a thing 14
of (sensible) size is put at the head of the gnomon/ 15
so that a shadow appears for it on the ground. It
will be seen that its shadow is small,/ and distinct 16
from that of the gnomon. So for safety one should
make for the conical head of the gnomon/ a sphere 17
which, when put on it, (casts a shadow) on the ground
of the size of a chick-pea. It is pierced with a / 18
conical hole so that if the head of the gnomon pene-
trates it, its tip will reach/ its center. The sha- 19
dow of this sphere will be found on the earth, quite
distinct from the gnomon's shadow./ But it is not 19:1
very distinct on the shaded area from the foot of
the gnomon to the center of/ that shadow which re- 2
sembles the chick-pea. The greater the altitude of

[1]Text تثير ; read تنير as in the MS.
[2]Text والحق , MS وعق , read ويحق .

the sun the more/ the distinctness and the more 19:3
obvious the harm to (the results of) the operation.
It is more harmful in the case of the lunar shadow/
if it is used in any one of the operations, except 4
(those) of an approximate nature. If/ approxima- 5
tion is sought for (this) harm must be accepted
along with it.

Thus, let *ABG* (Figure 5) be a quadrant of 6
the circle of altitude with center *E*,/ which is 7
(the center of) everything, and *AE*[*H*][1] is in the
true horizon, and *T* the head of the gnomon,/ and *W* 8
its base at the locality, on the face of the earth,
and [*N*]*W*[2] in the/ apparent horizon. Let the sun be 9
at point *B*, its computed altitude (being)/ *AB*, and 10
its distance from the zenith *BG*, and we extend [*B*]*T*
[*N*][3]. So/ *WN* will be the shadow of gnomon *TW* at 11
this altitude, I mean the direct/ actual (shadow), 12

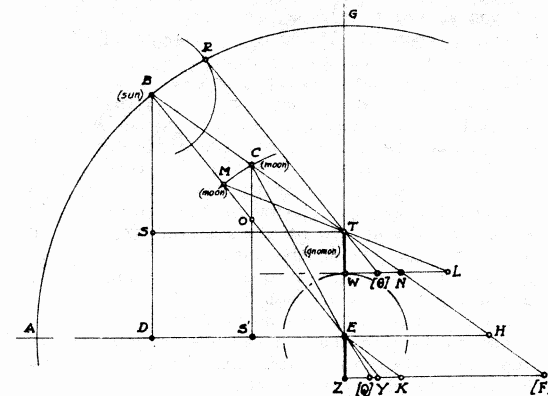



Figure 5

[1]Text has *AEG*. In Figure 5 the text has *B* for *F*, *N* for *Q*. See Section 19 in commentary.
[2]Text has *ZW*.
[3]Text has *NTZ*.

and we [pass][1] *EZ* equal to *TW*,// and we f.197b
extend *ZK*/ parallel to the horizon intersecting 19:13
hypotenuse *BE* at *Y*. So *ZY* would be the shadow
of/ the gnomon had its head been the center of 14
the earth. And because angle *WTN*/ is external to 15
the triangle *ETB*, it will be larger than angle *TEB*/,
which is opposite to angle *ZEY*. So angle *ZEY* is 16
less than angle/ *WTN*. Let us make angle *ZEK* equal 17
to *WTN*. So/ triangle *ZEK* will be equal to triangle 18
WTN and *ZY*, the shadow at/ the center, will be 19
smaller than shadow *Z*[*K*][2] at the locality. And so
the ratio of the gnomon to its shadow/ at the 20:1
locality will be smaller than the ratio to it at
the center, except that the amount of *ET*, which/ 2
is the distance between the center and the locality,
compared to *EG*, which is the distance of the sun/ 3
from the center of the earth, [is less][3] than a
half of a tenth of a sixth of a tenth of one, ap-
proximately,/ I mean three minutes of the total 4
sine, taking it as sixty parts, and it is the dif-
ference (upon subtracting)/ *BS*, the sine of the 5
altitude from *S*, from *BD*, its sine [as computed][4]./
One does not find for it, by consideration and 6
examination, any perceptible size, and especially
with altitudes/ which exceed an eighth of a revolu- 7
tion. If it had any effect, then there would be
found also between two shadows of/ two different 8
gnomons at one altitude a perceptible difference.
That is because/ the greater of the two, if it is 9
TZ and the smaller *EZ*, the shadow of the larger
would be/ *ZF* and the shadow of the smaller *ZY*, and 10
the ratio of *TZ* to *ZF*/ is the ratio of *EZ* to *ZK*, 11
and it is smaller than the ratio of *EZ* to/ *ZY*. 12
But in actuality the two are equal and the dif-
ference which is caused by the amount/ *ET*, in 13
whatever has to do with the sun and what is above
it, is imperceptible with instruments,/ but should 14
rather be extracted by computation.

---

[1]Text نغزز ; read نغرز as in the MS.

[2]Text has *ZY*.

[3]Text يقصد ; read يقصر as in the MS.

[4]Text محسوسا ; MS محسوا , read محسوبا .

The only heavenly bodies which cast 20:15
shadows with gnomons/ by their rays are the sun 16
and the moon. For Venus, even if anything like/ 17
that were to be found for it, it would not be
complete, so that the shadow of the gnomon from
it could be perceived. But it has/ a light inside 18
darkened houses, if it shines through holes in them,
(casting) what seems/ to be a very faint shadow. 19
But Jupiter in this respect/ is much weaker. 21:1

So let us lay down *EM* (Figure 5) equal to 2
one of the distances of the moon from the center,
and let it be its nearest distance./ Let the moon 3
be at point *M* at the part (of the ecliptic?) where
the sun/ has the same azimuth by computation. It 4
is based on the angles which are at the center of
everything. Let its shadow/ be [*W*]*L*[1] and let the 5
ratio of the gnomon *TW* to it be less than its ratio
to *WN*, the shadow of/ the sun when it is in this 6
position, I mean since the distances of the two from
the zenith are/ one amount. And we describe about 7
center *E* and [with][2] distance *EM*, the amount of the
moon/ in [its][3] sphere, arc *MC*. If the moon is at 8
point *C* its shadow/ at the locality will be *WN*, and 9
at the center (of the earth) *Z*[*Q*][4]. So the ratio
of the gnomon to the shadow of/ the sun at the cen- 10
ter will be less than its ratio to the shadow of the
moon at it. And the ratio of the gnomon/ to the 11
shadow of the moon at the locality will be less than
its ratio to its shadow at the center, and/ dif- 12
fering more than that of the shadow of the sun at
either of them. But the amount of *ET* compared to
EM/ is close to [a thirtieth][5], I mean two parts of 13
the total sine. It will undoubtedly produce/ as the 14
result of subtracting *CO* from *CS'* a difference which
is perceptible to the senses. And the angle of the

---

[1]Text has *EL*.

[2]Text نبعد ; MS سعد , read ببعد .

[3]Text كرية ; read كرته as in the MS.

[4]Text has *ZW*.

[5]Text has ثلاث عشر , thirteen.

complement of/ the computed altitude at the 21:15
center, I mean *GEB*, differs only from the one
perceived/ by sight, I mean [*G*]*TB*[1]. As for the 16
one (obtained) by sight, for example, which is
the case when the moon is at/ *C*, its distance from 17
the zenith will be observed as the distance of the
sun when it is at/ point *B*. Their two shadows at 18
that time will be to the amount of [*WN*][2], and the
amount by/ which the lunar parallax, in its heaven, 19
has changed, is *MC*. (Verily) there is no planet/
which does not have a parallax, but it differs 22:1
according to its distance/ from the earth. 2

Since the determination of the solar paral- 3
lax by instruments is difficult,/ its effect on 4
shadows and altitudes is small, hence its different
situations/ at the apogee of its orbit or its perigee 5
are imperceptible, otherwise the ratio of the shadow
to the gnomon at/ the// apogee would be less f.198a
than it (is) at perigee. 6

For this supposed reason, which is not real, 7
al-Kindī said that the shadow/ at the head of Aries 8
is less than at the head of Libra. He should
have mentioned as a condition for it/ its time (as 9
being) due to the motion of the apogee. However,
in the case of the moon, because of its nearness to
the earth/ the amount of its parallax at its extreme 10
value exceeds one part (i.e. degree). So its effect
is apparent and is observable/ by instruments, and 11
it is effective in eclipses so that there will be
between the/ hidden (i.e. true) conjunction, which 12
is that when the two luminaries are at *B* (and) *M*,
and the/ apparent conjunction, which is (when they 13
are) at *B* (and) *C*, an amount of time/ which is 14
perceptible. The greatest distance of the moon
from the earth is close to the double of its/ 15
nearest distance, so that there is a great differ-
ence between the moon's shadows at its two dis-
tances. This (situation) and things like it are
among/ the just causes which make the practitioners 16

---

[1] Text has *HTB*.
[2] Text has *EG*, the MS has *EG*; see the commentary.

of this art dispense with the use of lunar 22:16
shadows in what/ they need. So that they have 17
forsaken it entirely, except (when driven) by
necessity, since it/ leads to other than what 18
should (be found), and runs to [estimation and
guesswork][1].

However, as to the saying of al-Kindī that 19
if we erect a gnomon, we find its noonday shadow/ 23:1
from the sun to be less than its shadow from the
stars, due to the width of its body, (we say)/ 2
had they been wider their shadows would have been
shorter.

So let, as he said, half the (apparent) 3
diameter of the sun be *B*[*R*][2], (Figure 5) and ex-
tend *RTΘ*./ Then *R* is the edge of its body with 4
light (centering) at *B*, and so [*Θ*][3] will be the
extremity of its shadow,/ whereas that from its 5
center must be at *H* (read *N*). There is no use in/ 6
mentioning the planets, because there is no shadow
from them, but we should have said that the/ actual 7
shadow will be shorter than what is demanded by
computation. Also the mention of/ noon is redun- 8
dant, since this condition is common to shadows
at all times.

---

[1] Text النجمن والجرر ; MS النخيين والخرر ; read التخمين والحزر.
[2] The text has *Z* instead of *R* throughout.
[3] Text has *O*.

THE SIXTH (CHAPTER) 23:9

ON THE METHOD BY WHICH/ THE USE OF THE SHADOW AND THE GNOMON IS ARRANGED 10

The non-transparent parts of the earth protruding from the planes parallel to the horizon/ will have shadows, like the shadow of the earth, when the sun rises,/ in a direction opposite to that of sunrise. 11 12 24:1

Abū Zaid al-Balkhī mentions, concerning aspects of the usefulness of mountains, what I am going to relate/ word by word. He said: "One of the aspects of the usefulness of mountains is that they cast shadows and provide cover, and that is that had/ the earth been clear, with no curtain on it from the sun all day,/ no life would have endured, and there would have been nothing to make forming and reproduction possible. For then/ all of these would need shade to protect them from the blazing of the sun, much as they/ need to bask in the sun, seeing that they cannot dispense with the shining of the sun/ on them. 2 3 4 5 6 7 8

"The shadows which shade animals and plants are of two sorts: the shadows of/ trees and the walls of dwellings, and the shadows of mountains. And it is evident that the shadows of/ the first kind are less useful than the other for two reasons. 9 10 11

"One of the two is that they are wide, and the shadows of mountains are constant shadows, not declining,/ and the second is that the powers of the shadow of any object, to protect from either heat or cold,/ is proportional to the thickness of the parts of the intervening object casting the shadow and its width./ Hence the shadows of lofty mountains are beneficial to animals and plants, in/ reality, but not the artificially (produced) shadows. 12 13 14 15 16

And because of this,/ valleys are refuges for those who dwell in them, and hiding places from heat and cold, and that is why God, be He exalted!,/ counts them among his blessings. And so He said:[1] 'Of what He created for you, He created (also) shadows'". 24:17 18

Abū ᶜUthmān// al-Jāḥiẓ said: "The shadow of the stone is black". For/ the denser the intervening object the more intense its shadow in blackness. The Arabs say, "There is no shadow like that of/ a rock in intensity, and no warmth like that of a tree". Nothing will be cooler nor more intense in blackness/ than the shadow of a mountain, and (more) especially the wider and higher it is. And this is why one retires/ from the rays mingling with the edge of the shadow, regardless of the denseness (of the mountain). f.198b 25:1 2 3 4

However, as to what is mentioned about the warmth of trees, it is due to the resistance of the crevices, like the resistance of/ any covert against it, and a cave in a mountain is more effective than that, because there are no leaks. But/ to see warmth in trees at the time of their combustion, and to ascribe it to them as a matter of the fire latent/ in them, is to deem it itself resident in them. Verily they say that the shadow of shadows is/ the shadow of a crevice, and the shadow of the *tanᶜīma* (a thorny tree with fleshy leaves), and the shadow of a rock. The analogy is based upon the thickness of the/ leaves of the *tanᶜīma*. Indeed it is said that it is like the Swiss chard. Thus they are associated with the stone. And everything which is associated with a shadow/ should be shared by the stone and the tree, according to their forms. 5 6 7 8 9 10 11

Then we say that plane surfaces upon which shadows fall/ are numerous. They are all planes of local horizons which will be determined if their latitudes are known./ And the shadows of gnomons for their altitudes are known, and in spite of that they are of/ two kinds like (any) category containing its (different) types. One of the two of them is the 12 13 14 15

---

[1] Qur'ān 16:81.

shadow which/ we have been discussing up until 25:16
now. It is bordered by the shadow-caster itself
and the ray/ passing from the sun to the head of 17
the shadow-caster, and that part of the horizon
plane which is between them. But/ since light 18
can be perceived on the flat of the earth, a place
devoid of light is called/ shadow, and the shadow- 19
caster is called the gnomon (*shakhs*), but when it
is used, expecially in computation, (it is called)/
the scale (*miqyās*). 26:1

That sort of shadow is always in the plane 2
of a circle of altitude/ through the shadow-caster 3
(and cast) on the part in common between it and
the plane of the horizon in the case of the vertical
gnomon perpendicular to it./ It is called the ex- 4
tended (shadow *or* cotangent) because its extension
is along a face of the earth which has neither pro-
trusions nor concavities.

Thus is the horizon plane, and the inclina- 5
tion of any other/ plane surface is non-zero, except 6
for those perpendicular to it.

An example of the direct shadow (*or* cotangent) 7
is (the following): Let *A* (in Figure 6) be the body
of the sun and *BG* the gnomon/ perpendicular to *EG*, 8
which is parallel to the horizon plane, and *ABE* is
the sun's ray/ passing through the head of the gno- 9
mon *BG*. So *BGE* will be/ the shadow in space. But 10
EG is that which is called the direct shadow such
that its base is/ *G* and its end *E*. And *EB*, the 11
line joining the two ends of the shadow and the
gnomon,/ is the hypotenuse of the shadow (*or* the 12
cosecant).

However, as for the second type of shadow 27:1
it is that whose gnomon is [parallel to][1] the
horizon plane./ Then the gnomon is perpendicular 2
to a plane which is itself perpendicular both to/
the horizon plane and the circle of altitude. And 3
the shadow itself (accordingly) will be along the
axis of the horizon./ It is called the reversed 4
(shadow *or* tangent) because its head is under its
base, and it is called also the erect (*muntaṣib*)/
because it is erected along that diameter of the 5

[1]Text وارى ; read وازى as in the MS.

(terrestrial) sphere through that locality accor- 27:5
ding to/ this example[1]. 6

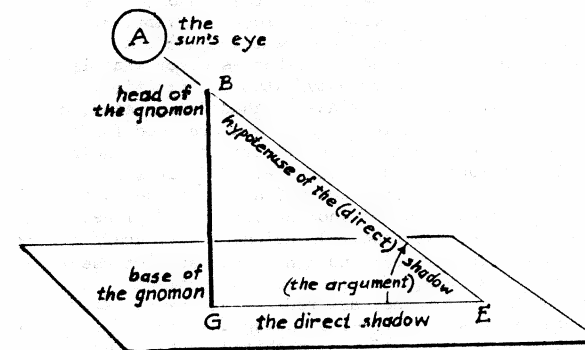



Figure 6

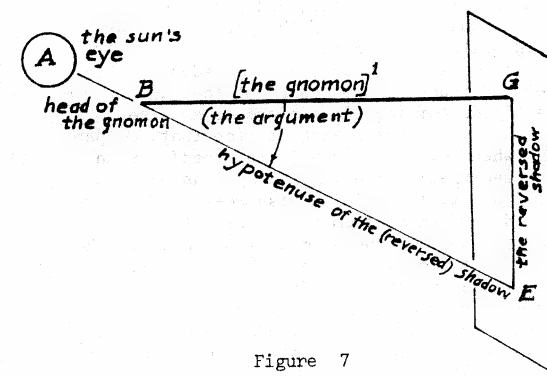



Figure 7

[1]Figure 7 illustrates the situation, although there is no specific reference to it in the text. Text: الشمس ; read الشخص as in the MS.

The saying of the Ṣūfīs is hardly understood among them(selves), much less/ among others, and especially the word of al-Ḥusayn b. Manṣūr al-Ḥallāj. He says/ in "The Book of Red Sulfur" (*Kitāb al-kibrīt al-aḥmar*) that the shadow of the vertical (gnomon) is (itself) erect and vertical, and that the other/ shadows are low (and) flat (*munbasiṭa*). Thus (he) describes as erect what is inclined, which is a grossly/ stupid definition. These two types of the kinds of shadows are studied and applied/ in all localities, and are used in common in the determination of altitudes and their complements and are associated with/ their sines. 27:7 8 9 10 11

However, as for that which is outside these two classifications, of gnomons set up on/ planes other than the two (above-)mentioned, they are// f.199a unlimited (in number), and hence they are not used/ except for showing off with some astronomical instruments. But they are reduced at the end/ to one of those two types by being associated with one of the horizons of the inhabited earth. Indeed we have said/ that they are shadows for altitudes at horizons coplanar with their planes./ We need for them, first of all, the determination of the inclination of those planes, and then (we need) some ingenious operations,/ the performance of which is quite troublesome. 28:1 2 3 4 5 6 7

It results from what has preceded that the apparent altitude in the sphere of the sun is/ the true altitude for it, since there is no size to the earth in comparison with it, (so that) in particular whenever the shadow is measured/ for an assumed zenith-distance for the sun, as well as computed at it, there is no disagreement./ There will be disagreement from the excess of what is between the head of the gnomon, whose head is the/ apparent center of the universe [over the true center of the universe][1]. It (the difference) is due to its (the apparent sun's) appearing below the (true) sun. The two altitudes, the apparent and/ the true, differ. If that is considered for 8 9 10 11 12 13

[1]The added words seem required by the context.

such lights and/ luminous objects as may be below the moon, it will be indeed very great in magnitude. 28:14

From this it is clear that regular procedure in the use of shadows with regard to/ the rays of the sun should be by taking the head of the gnomon as the center of the earth, and in the/ use of rings (the armillary sphere?)[1] and it is by taking its center as its (the earth's) center, and that is different/ from the actual source, the sun. 15 16 17

[1]Here a word has been effaced in the MS.

THE SEVENTH (CHAPTER) 29:1

ON THE DIVISIONS INTO WHICH GNOMONS ARE DIVIDED

Necessity demands the disappearance of shad- 2
ow in the region in which it extends, when/ the 3
amount of the illumination exceeds the amount of
the shadow in capacity. And in what remains (other
than this) there will not be/ any shadow which ends 4
at the (borders) of its area of extent. But we
have shown that shadow and illumination in/ a trans- 5
parent (medium), of real transparency, are alike,
since they are perceived at its boundary/ with a 6
broad, non-transparent object, so that there appears
of it what faces the source, while that part covered
by the shadow-caster is darkened,/ on the plane 7
through the shadow, the source, and the intervening
(object) between the (first) two. Such is/ the sit- 8
uation with the shadow of the earth, for it is in
the air, extended, surrounded by light, and we do
not/ sense either one of them, except on the moon if 9
it or a part of it penetrates the shadow, so that it
eclipses that (part) of it/ which enters the shadow. 10
The rest remains outside it, lighted. By this
we perceive the circularity of/ that shadow. It is 11
an indication of the roundness of the earth, because
that shadow will be/ similar to the common part bet- 12
ween what is illuminated of the earth, and what is
darkened of it. And our finding/ that shadow in 13
lunar eclipses, to be circular of edge, (together)
with the different positions of/ that common portion,14
as to the earth's longitude and latitude demon-
strates/ its roundness, and that the protuberance 15
of mountains does not affect it, since they are small
in comparison with the great magnitude of the earth.

Since the situation is thus, we say that 16
the altitude of the source of light from the hori-
zon/ is the same with respect to the gnomon set up 17
on its surface as the excess of the amount of 29:17
the illumination over/ the shaded object, and 18
that is why its shadow has a finite magnitude,
determined either by perception or computation./
So if the source has zero altitude, this would 19
imply for the gnomon the equality of the source
and the shadow-caster,/ and thus shadow will not 30:1
have an end on the side [opposite] which it has
altitude, I mean from the side/ of the end, as 2
is the situation at sunrise and sunset, for the
shadow will then be/ infinite. 3

Hence if it is from a small hole behind 4
which there is a lamp, with the top of the/ hole 5
(along the) parallel to the head of the gnomon,
the shadow would extend indefinitely./ And if 6
the end (of the shadow) disappears at equality,
then it will *a fortiori* disappear if/ the source 7
goes below the end of the gnomon, since that is
similar to the lessening in the amount of the
source as compared to/ the amount of the shadowed 8
part. Hence the actual shadows of gnomons set
up on the face of/ the earth will depend in 9
length and shortness on the altitude of the
source in smallness/ and greatness. I mean that 10
the shadow will decrease with increase in the
altitude of the source until it is reduced to
nothing/ upon the altitude reaching its extreme,//
beyond which it will not increase, which is f. 199b
its reaching the top/ of its head, as is related 12
by a reciter:

> When[1] the caravan leader urges 13
> the jaded beast,
> And the shadow shortens, becoming
> (like) a sock.

And another:

> When the saddle beast tires its
> driver,
> And its [soles][2] ride over its
> neck.

---

[1] Text ازازفالحادى ; MS ازانّ في الحادى .
[2] Text احفاقها ; read احفافها .

As Abū al-Najm says, "The shadow does not exceed its soles."/ Some tried to beat about the bush to find a reason, so they make it the intensity of heat. They said: 30:16, 17

Noon has its heat enkindled. 18
I made my eyebrow (*ḥājibī*) a curtain (*ḥājib*),
his only protection against it.

One flees the sun seeking a (safe) place. 19
(Even that) of the seeker's open enemy.

The chameleon prostrates itself before the sun 31:1
As the priest bows before the monk.

Near to them is he who said: 2

How often the midday heat 3
Is [unbearable][1] for caravans.

While the sun devours its (own shadow, 4
(Just) as fire eats [the wood-cutter's][2]
[sticks][3].

But the shadow increases with decrease in altitude until it reaches its extreme/ beyond which there is no extreme, I mean its being nonterminating when the altitude vanishes. 5, 6

The situation with the reversed (shadow) is opposite this, for the shadow is necessarily a quantity straight/ in portrayal if the gnomon is straight. And if one operates with an arc-shaped instrument, then/ finding the corresponding arcs in making it is not satisfactory at all. The altitude is along an arc of a circle./ Proportions between arcs and straight lines are unknown, and not subject to the/ methods of the known ratios. Hence they are between the sides of the triangle 7, 8, 9, 10, 11

---

[1]Text تفل بحرهاصير ; read يقلّ بحرهاصبر .
[2]Text حاجب ; read حاطب .
[3]Text عصرتاب ; read عيران .

determined by/ the gnomon and the shadow and its hypotenuse, and the sides of the triangle determined by the sines of/ the altitude and its complement, and the maximum sine, because they are straight lines and the triangles are/ similar. And because the gnomon is what makes the shadow, and is fixed in its amount in spite of the/ change in amount of what it makes, the shadow is measured by it, especially if it is/ referred to when used for time-determination [or][1] in astronomical computation./ In the determination of the times of prayer, the shadow will not always be equal/ to the gnomon, or an integer number of times its length. So sometimes it will be part of the gnomon and sometimes/ a multiple of it plus somewhat less than the whole of its length. So there is a need for graduating the gnomon/ in parts so that the amount less than the equal (of the gnomon) can be measured in order to put that (as) the ratio between/ two numbers, as needed with the other quantities utilized as units for measuring/ weights, capacities, lengths, and so on. 31:12, 13, 14, 15, 16, 17, 18, 19, 32:1, 2, 3

And since the number of these parts is not something naturally imposed, but rather/ put (arbitrarily), those who use it differ, since they do not belong to a (single) place or time or opinion./ Each one of them chose arbitrarily (a number) as a guide, which does no discredit to the operation, as long as/ it is retained (for consistent use) by one man, or it becomes known and applied by many people together. 4, 5, 6, 7, 8

But, if (a certain number) is preferred, for some purpose, either from habit or by imitation,/ it should be treated as (described for) the first (mentioned) either by one (individual) or by many. 9, 10

As to what we have found of opinions in our time concerning the number of these/ divisions, there are three types. One is sixty, and it is the opinion of the people of the West (*ahl al-maghrib*), being/ used in the book Almagest, and in the zījes of the followers of Ptolemy and the Greeks/ and the moderns. The reason for it is that he took the half-diameter of the circle, when he wanted to extract/ 11, 12, 13, 14

---

[1]Text اذ ; read او as in the MS.

the ratios of the shadows to their gnomons at the 32:15
two equinoxes and the two solstices, as a gnomon
and/ he made half the diameter of the circle sixty 16
parts. So the gnomon was graduated/ in its (the 17
radius') parts, and the moderns follow him and
profit from it in two ways.

One of the two is that some of their opera- 18
tions with shadows become much easier than/ what 19
was previously done, by (using) sines, and there-
by they were relieved of half the difficulty.

The second is that this number is the (num- 33:1
ber) of parts in one (unit) among/ the astronomers 2
and among many of the people concerned. For multi-
plication by/ the total sine and division by it 3
becomes easier, being (performed) by depressing
its rank to minutes or elevating it/ above them. 4
Thus also is multiplication by the gnomon and di-
vision by it, or multiplication by// one of the f.200a
two/ and division by the other is thus simplified. 5

I added to the operations in the zījes the 6
merit of (additional) simplicity by making both/
the total sine and the gnomon one part, so that 7
there fell from them the need for depression and
elevation/ completely. 8

The second type of the number of the divi- 9
sions is twelve, and it is the opinion of/ those of 10
the East (*ahl al-mashriq*), the Indians being [among][1]
them. For they characterize the latitudes of local-
ities by the shadows at/ the equinox and the two 11
solstices. And they perform most operations with
shadows, and they call the/ parts of these divisions 12
digits, in their language [*ankula*][2]. Thus in the
Arkand Zīj/ the digits and their minutes are (put 13
as) [*anjula*][3] and [*bīanjula*][4]. But I have never
heard of (such?) minutes,/ rather he carried over 14
the form of the name as it was in the copies.

---

[1]Text نبيهم ;read بينهم as in the MS.
[2]Text الكل ;read انكل as in the MS.
[3]Text انحل ;read انجل as in the MS.
[4]Text ينحل ;MS يتنجل ;read بينجل.

The reason for these twelve divisions 33:15
happening to be called digits is/ that a normal 16
span contains twelve normal digits, because (it is)
three/ hand-breadths, the hand-breadth being four 17
digits. The magnitude of the span falls between
large (units)/ which are (too) large, and small 18
units, which are (too) small. Moreover it accom-
panies man most frequently (whether)/ in travel 19
or in sedentary (life), in contrast to common
metallic (objects) such as knives, rulers,/ awls, 34:1
pegs, and (things) like them used [for][1] mea-
suring shadows, which are (therefore) not consid-
ered in most/ circumstances. So he who needs to 2
measure a shadow begins, in the (last named) situa-
tion, by setting up a knife (which he has)/ with 3
him, or he makes a peg resembling these two (i.e.
the span or the knife). The custom as to the size
of knives (is that) they should be like the knives
of/ the virtuous, not (like) the daggers of the 4
malefactors, which are about a span, or what is
close to it either/ bigger or smaller. So, if 5
the (thing) set up is an (actual hand-)span, and
if then the shadow is measured (with the hand) in/
the gnomon (length) then the part (remaining), which 6
is less than a gnomon length, can be measured in
hand-breadths and digits, (thus) determining the
shadow./ So a half of a sixth of the gnomon is 7
called a digit. Many (times) I used to witness
the Indians,/ if they wanted to ascertain the time 8
for their operations, which will be explained
later, they would pass their hands/ in the direc- 9
tion of the sun so that the hand, from the vicini-
ty of the elbow, became parallel to the horizon,/
with the inside of the forearm and the arm toward 10
the sky. Then they would erect half the middle
finger/ so that it became a gnomon, and its shadow 11
extended itself along the inside of the palm and
the forearm. Then they measured it in digits/ of 12
the other hand. But it did not occur to me to ask
them about what follows that in their operation,
but they/ must have multiplied these digits by four 13
in order to make them real digits,/ because the half 14

---

[1]Text بل قايس ; read بقياس .

of the middle (finger), the shadow of which they 34:14
measured is a quarter of a span, which is three
digits./ And the ratio of the three digits to 15
their shadow is as the ratio of twelve to its
shadow.

So the truth of the transformation is that 16
we multiply the digits of the actual shadow of half
of/ the middle (finger) by twelve and we divide the 17
result by three. There come out digits of the shadow
of a span/. If we change (it) the ratio of three 18
to twelve would equal the ratio of the actual shad-
ow to/ the required shadow, and three is a quarter 19
of the twelve. So the digits of the actual shad-
ow are always a quarter of the/ digits of the 35:1
required shadow.

It is possible for them to postpone the 2
transformation until later./ For they multiply, 3
in the course of the operation, something by four
times what is needed, or they divide/ something 4
(else) by a fourth of what is needed, whereupon
the transformation results.

(Analogously) to what we have mentioned, a 5
part out of twelve, for each/ of the diameters of 6
the two luminaries is called a digit, because each
of them is, in the midst of the sky, a span/ in 7
appearance (i.e. in apparent diameter).

However, in the Arkand Zīj that part is 8
called by a short name in/ their language, it being 9
māshah, (and) not digit. Each *māshah* is four *kākī*,
but I have never heard/ this latter name. As for 10
the first, although I have heard it, it was in
(connection) with weights./ For they call a weight 11
of three gold dirhems a *tūlah*, it being twelve
māshah,/ and each *māshah* is four *wandī*, and each 12
wandī is four *jawa*, which is a barleycorn.// f.200b

When the ratio is established as we men- 13
tioned, the ratio of the shadow to the shadow-
caster/ for a single time and at a certain place 14
will be a fixed ratio./ The parts into which the 15
shadow-caster is divided into twelve divisions,
(whether) large or small, are called digits.

The third variety is the seven, or the six 16
and a half. Each/ of these two (units) is called 17

a foot, it being the opinion of the Muslims bet- 35:17
ween the (above-)mentioned opinions./ The reason 18
for it is that they did [not][1] need (to use) shad-
ows for what the Byzantines (*al-Rūm*)/ and Indians 19
needed them, but rather they needed the noon shad-
ows in order to ascertain the time of the after-
noon (prayer)/ due to the necessity of adding (to 36:1
the noon shadow) in order to maintain the prayer
for its time can easily be confused./ Because 2
those appointed[2] to determine it are the muezzins
of the mosques, and those of them who seek veri-
fication/ imitate the opinions of the astrologers 3
as to instruments which they made and set up for
them/. They (the muezzins) add their own pro- 4
fessional deductions, thus fixing the magnitudes
of the/ noon shadows for their localities for all 5
the days of the year by examination and considera-
tion until they reach/ the extraction of the time 6
of the afternoon (prayer) from them. So they took
the heights of their own bodies as gnomons, since
these are/ natural columns. They associated with 7
them the shadows fixed by them. But they needed/
to measure the shadow, and the foot was the nearest 8
(thing) for it, because it is an old (procedure)
and it is a custom among the people/ to measure 9
the sizes of houses in feet when they lay down the
foundation of the wall, and take/ measurements for 10
their carpets and furniture and things like that.

The normal foot is to the normal height of 11
the same person in a/ known ratio. They state that 12
it is the ratio of one to seven, and (even) as/
one digit is half a sixth of the gnomon, so (also) 13
the foot is a seventh of its gnomon, and the/
sevenfold divisions of that they call feet. 14

The other of the two parties (of muezzins) 15
are of the common people, whose hearts are dis-
gusted by the mention of shadows,/ or altitude, 16
or sines, and who get goose-pimples at the (mere)
sight of computation or (scientific) instruments./
With them it reaches such an extent that one can- 17
not trust them with anything or the sort, much less

---

[1] Text لما ; read ما.
[2] Text المتدربين ; MS المتدرين.

the times/ of prayer, not because of unfaith- 36:18
fulness or treason, but because of excessive
ignorance.

An example of it is that one of them sought 19
my advice on this subject, and impelled me,/ be- 37:1
cause of his great ignorance of his profession,
and my being afraid that he will make a mistake
in the rules of my religion, to save him/ from 2
guesswork by the use of an instrument for (deter-
mining) the times of the two prayers of the day
according to the doctrine/ which he held. I 3
showed him the Byzantine months, substituting
(them) for the names of the signs./ Then he be- 4
gan to suggest about it that it should be made
according to the Arab months. So I stated to him
that the matter/ has nothing to do[1] with them, 5
and in addition to being very confusing they would
require intercalation,/ which is forbidden in Is- 6
lam and very heretical. But his ignorance made
him at the end/ refuse to accept anything based on 7
the Byzantine months, not allowing it/ into the 8
mosque, since (those) people are not Muslims.
Then I said to him: "The Byzantines also eat food/ 9
and walk around in the market. Do not imitate
them in these two things". And, when explanation
and/ instruction were useless to him, I confronted 10
him, after all the stupidity, with (the fact of)
his disease for which there is no cure, then I
saw [him][2] forsake/ the reckoning by breaking that 11
instrument, [which sufficed him[3](?)].

As for those who take six and a half feet 12
for their gnomons,/ it was suggested by (a desire 13
for) precision, namely that for someone measuring
the shadow by his own height it is impossible to
turn away from/ the sun's eye and face the shadow 14
in order to observe its amount and to put a mark/
wherever its end (point) reaches. For by neces- 15
sity the great toe of his foot is in the direc-
tion of its (the shadow's) end, but/ the side cast- 16
ing the shadow is that of the face, and that which

[1] Text يسنبهم ; MS يلبسهم.
[2] Text رأيت; read رأيته.
[3] Text حسبة ; read حسبه.

makes its end is what/ protrudes from the head 37:17
at the forehead. But the plumb line (held) by
the front of/ his thumb falls away from the heel 18
in the direction of the toes, and the foot of
the plumbline will be at the/ center of the sole. 19

So if the shadow of the one standing is 38:1
seven feet// at the time when the shadow of/ f.201a
anything is equal to itself, and if the begin- 2
ning of the seven is from the heel, (which is)
behind the/ (above)-mentioned plumb line, then 3
there enters into the shadow exactly half of the
foot from the (side of the) toes, and there
remains/ the half (on the side) of the heel, which 4
is not counted. Thus there remains, after this,
the shadow of the front side as six feet/ and a 5
half, and they are feet of the plumb line, because
we assumed the time to be when the shadow equals/
its gnomon. 6

Hence it is prescribed in some of the books 7
not to count in the shadow the foot of the one
standing,/ when measuring it. The prescription 8
in both sayings is an unnecessary refinement and
hence/ it is valid for someone else also standing[1] 9
(to take his shadow) to increase it, and to express
his opinion saying, "The head,/ no matter how dif- 10
ferent from the normal shape it may be, is like
the inhabited sphere on two sides"./ Except that 11
some tribes of people alter God's handiwork, like
the people of Khwārazm. They/ flatten the heads 12
of infants, broadening (them) by pressing them in
the cradle from front and/ back, making them sub- 13
ject to [rebuke][2] and warning by the people of the
world. It is possible that there are/ in the world 14
others like them.

Hippocrates mentioned in the book "Climates 15
and Countries" that (some) people/ broaden their 16
heads, taking pride in bravery, so that broad heads
became/ a (distinguishing) mark for them. But there 17
occurred to him an impossible notion about them.

[1] Text الموقف; MS المدقق.
[2] Text مثله ; read مثلبة.

So he said that the nature of that arti- 38:18
ficial act of theirs goes into the offspring,/
they being born with broad heads naturally. But 19
Galen (rightly) criticized him for this.

Like the people of Farghāna who press the 39:1
[top][1] of the head so that/ the foreparts of 2
their heads overlook the forehead, and this be-
comes (the part) causing the end of the shadow,
and for these (people)/ the plumb line from their 3
foreheads indeed falls to the middle of their
feet./ So the shadows they cast, according to 4
the preceding law, are six feet/ and a half. 5

However the Khwārazmians need to make it 6
seven full feet/ because the highest parts of their 7
heads, between the [temples][2], are the parts that
cast the end of the shadow beginning from/ their 8
heels, at which the plumb line from them (the tops
of the head) falls.

However, as for heads left in their original 9
nature, the highest place/ on them is the vertex (of 10
the cranium), and the plumb line from it falls at
a (point) one third of a foot from the side (towards)/
the heel. So it is necessary that its base be six 11
feet and two thirds of a foot, halfway/ between 12
that of the Khwārazmians and (those of) Farghāna.

One of the most wonderful things is that 13
Abū Maʿshar fixed the shadow in feet in the/ tables 14
of his zīj at six and two thirds, but he used it as
six/ and a half. And al-Nayrīzī and Muḥammad b. ʿAbd 15
al-ʿAzīz al-Hāshimī took it over into their zījes/
in the same way. 16

Al-Hāshimī showed himself better than the 17
two of them in that he made the (bases) equal in
the table and the applications,/ thus avoiding in- 18
consistency. Al-Ḥasan b. al-Ṣabbāḥ made the gnomon
in his/ Mukhtariʿ Zīj seven feet and a half, but 19
perhaps that was (an act) of the copyist, for he
increased/ the seven by the same amount as the 40:1
decrease to six and a half from it (seven). This
(sort) of proliferation of notions was/ among the 2

---

[1]Text فرق ; read فوق as in the MS.
[2]Text القوجين ; read الفرين .

stupidities of someone opposing the *Bāṭini'īn*,/ 40:2
who are fond of silly burial rites, in their saying 3
that the human is seven spans./ So that oppo- 4
nent said, in refuting and contradicting them,
that rather it is eight spans, because/ the shirt 5
is six spans and a half, and the height of the hin-
der part (of the dress) from the ground is half a/
span, and the head with the neck is more than a span. 6

Thus spoke the authors of the treatises of 7
the Brethren of Purity (*Ikhwān al-Ṣafā'*), that the
height of/ a standing (man) is three of his spans, 8
with a detailed account of the dimensions of all
the organs, based/ on ignorance. For this saying 9
of theirs is the extreme of foolishness unless/
their authors intended to push two spans into one. 10
But to oppose ignorance one should kill it. So,
concievably/ the first (i.e. the eight-span advo- 11
cate) thought that a span is less than a foot by
an eighth. Then if he transformed the height from/ 12
the seven feet into// spans it would become f.201b
eight, but even though it is not seven spans,/ it 13
is seven if (measured in) feet. So may God con-
join the razors and/ the beards of these reasoners. 14
Both of the two types, of digits and feet, are used/
for the direct shadow (the cotangent) without nec- 15
essarily reserving (the use of) feet/ for fore- 16
noon shadows.

As for the reversed shadow (the tangent), the 17
parts of sixty may be used for it/ as we mentioned 18
previously, and I called them minutes when I made
the total sine one part.

It is possible to divide the gnomon into two 19
and a half parts, equal to the amount of the total
sine as/ done by Brahmagupta in the Khaṇḍkhādyaka[1] 41:1
Zīj, or fifty[-four][2] parts/ and a half as he did 2
in the Brahmasiddhānta, or fifty-seven parts/ and 3
a fifth [and][3] a tenth as was done by Āryabhaṭa in
his book, and Pulisa the Greek [followed][4] him/ in 4
his siddhānta.

---

[1]Text كندلانك ; read كندكاتك as in the MS.
[2]Text اوارهه اوخمسين ;MS اوارهه اوخمسين ; read اواربعة و خمسين
[3]Text خمس عشر ; read خمس وعشر .
[4]Text افتفاه ; read اقتفاه as in the MS.

But the use of the shadow in the computation of arcs was made by the followers of/ Ptolemy. They divided the gnomon according to the parts of the half-diameter in the/ Almagest, following his lead, since also he divided it thus in the fifth chapter of the second/ book. As for the Indians, and what their five *siddhāntas* contain,/ none of their operations in them indicate the necessity of dividing the gnomon into the number of the (total) sine, and hence/ they did not know it nor did they come across it. 41:5 7 8 9 10

THE EIGHTH (CHAPTER) 41:11

ON THE TRANSFORMATION OF THE TYPES OF SHADOW

(OR TANGENT FUNCTIONS), ONE INTO ANOTHER

The types of shadow according to the parts of the gnomon are four: They are those measured/ in parts, in digits, in integer feet, and fractional. And if it is required (to find) each from/ each one of the three remaining, there result twelve couples./ The first of them is the association of the kind having parts with each one of the three remaining kinds,/ and that is three. Then the association of the kind having digits with each one of the remaining, and that is/ six. Then the association of the kind having seven feet with the three remaining, and that is nine. Then (is)/ the association of the kind having fractional feet with each one of the three, and that is twelve. 12 13 14 15 16 17 18

Each one of them (should be taken) as the direct and reverse (shadows), giving twenty-four,/ but that is unnecessary talk, void of meaning, since transformation concerns/ quantities and divisions and not what is produced by the erection of the gnomon and the plane of its shadow, since/ the feet have nothing to do with mention of the reverse (shadow), but rather it describes the shadow measured/ in parts more than the description of the direct (shadow), except by necessity. 42:1 2 3 4 5

As for that measured in digits, it falls between the two properties, at its condition resembling/ equality, and the property of the direct (shadow) predominates in it. So, to enumerate the twelve associations/ after speaking in general (terms, we say) that if the shadow is measured by one of the four (types of) divisions,/ and we wanted to transform it into parts of another kind, the ratio of the resulting shadow to the/ divisions 6 7 8 9 10

of its gnomon will be as the ratio of the desired shadow to the parts of its gnomon. The third of these/ four amounts is unknown, and one multiplies the first of these numbers by the fourth,/ I mean the resulting shadow, by the divisions of the gnomon of the desired shadow. What results is divided/ by the second, which is the (number of) divisions of the gnomon in the resulting shadow. And so there comes out the third of them, and it is the/ desired shadow.

On this (topic) Kūshyār b. Labbān laid down in his Jāmiʿ Zīj that one should multiply/ the assumed shadow by the divisions of the gnomon into which it is desired to transform it, depressed,/ that is divided by sixty, because the shadow put in his zīj is in parts of/ sixty. And everything which is divided by sixty will be depressed to what is below in minutes/ and seconds.

For similar reasons Abū al-Wafāʾ al-Buzjānī prescribed, both in his zīj and in his Almagest, that/ the parts of the gnomon which are to be transformed be multiplied by the assumed shadow./ He does not mention division because he made the parts of the gnomon sixty// minutes, and anything divided/ by one is the amount itself.

We take time to give a (numerical) example, beginning with the first coupling (of units), and say,/ suppose we had found the shadow to be ten parts and we wanted to convert it into digits. So we multiply/ the ten parts by twelve digits and divide the hundred and twenty by/ sixty parts. There come out two digits. We can postpone the division, as we said, or we proceed/ by dividing first the gnomon of the desired shadow, it being twelve, by the gnomon of/ the resulting shadow, which is sixty. And there comes out one fifth. Then we multiply this result/ by the resulting shadow, which is ten parts, and there results two. They are the/ desired digits. But this fifth is always fixed and unchangeable because of the constancy of the sixty and the twelve/ in their amounts. And it is known that the multiplication of an integer by a fraction

is [the taking][1]/ out of it (the integer) an amount whose ratio to the whole is as the ratio of that fraction to the unit. Thus it is necessary/ that we always take a fifth of the parts of the shadow, and thus it is transformed into digits. And because/ multiplication is easier than division, so multiplication of the parts by twelve minutes/ has the same effect as division by five for the taking of the fifth. The other transformations/ run along similar to this.

An example of it for the second coupling is that we want to transform the ten parts into/ the sevenfold feet, I mean with the gnomon of seven (feet). So we multiply the ten by seven,/ and we divide the seventy (resulting) by sixty, and there results one and a sixth. It is the feet/ in this shadow. If we begin with the division, dividing the seven by sixty there results seven out of/ sixty parts of one. If we multiply them by ten there result seventy out of/ sixty parts of one, and that is one and a sixth, as comes out in the first place, and the/ ease of the operation is not increased, since seven is not a factor of sixty; simplicity being a result of/ (their having) a common factor, and ceasing with their being relatively prime.

If we wanted to convert the parts into fractional feet, I mean (feet of) a gnomon of six/ and a half, [then][2] we multiply the ten by six and a half, and we divide the sixty-five by/ sixty, and there result one and half a sixth, and it is the feet of this shadow.

Their use, [converting a mixed number into a common fraction][3] is easier if we multiply the parts by thirteen,/ and divide the result by a hundred and twenty. Delaying the multiplication does not simplify the operation at all,/ since the result of the division of six and a half by sixty

---

[1] Text احر ; MS اخد ; read اخذ.
[2] Text ضربنا ; MS فضربنا.
[3] Text بجنسه ; MS مجنسة.

is thirteen parts out of/ a hundred and twenty of 44:14
one, and these two are relatively prime. Thus
if we took this half/ in minutes the gnomon would 15
be[1] six feet and thirty minutes. It is necessary/
that we divide three hundred and ninety by three 16
thousand[2] and six hundred minutes,/ and these two 17
numbers agree to the extent that each can be multiplied by a fifth of a sixth, giving/ thirteen 18
(parts) of a hundred and twenty. Thus the affair
reduces to what preceded, and there comes out six/
minutes and a half. And so if one multiplies by it 19
the ten parts, there result one (part) and five/
minutes, which is half a sixth, but the operation 45:1
does not increase the ease. (Now)/ the (number 2
of) couplings has been reduced by three. If the
resulting [shadow][3] of the gnomon is two digits,
and we want to/ convert it into parts we multiply 3
by sixty and divide the hundred and twenty (resulting) by/ twelve. There come out ten parts, and 4
it is the desired (thing).

If we had made the division precede, the 5
sixty would be divided by twelve to obtain five,/
whose product by the two digits would be the ten 6
parts. But the five is non-variable,/ and so (for) 7
the digits, multiplication is always by [five][4] to
transform them into parts, because the excess of
sixty/ over twelve is four times twelve, and five 8
times a thing equals/ the sum of it plus four 9
times itself. And so the digits of the shadow are
always as preceded, they will be a fifth of it in/
parts also. 10

If we wanted to convert the digits into the 11
seven-fold feet, we multiply them by/ the seven 12
and divide by twelve, and there results// a f.202b
foot and a sixth, and performing the division
first/ does not increase the simplicity because 13
seven and twelve are relatively prime. But if
we halve the twelve/ and increase its half by its 14

[1] Text فكان ; MS لكان .
[2] Text آلاف; MS الف .
[3] Text كان الشخص ; read كان ظل الشخص .
[4] Text ستة ; read خمسة .

(the half's) sixth the seven(-foot) gnomon re- 45:14
sults, and thus if we/ [halve][1] the digits of the 15
shadow and increase its half by its sixth, there
result the feet of this shadow.

It is known that increase by the sixth will 16
be by multiplication by seven and division of the/
result by six, because the ratio of any quantity 17
to the sum (of itself) when added to its sixth is as
the ratio of six/ to seven, and hence when we mul- 18
tiplied half of the [given][2] digits by seven there
resulted/ for our example seven, and by its divi- 19
sion by six there comes out a foot and a sixth, the
same as came out/ at first. Whether we divide it 46:1
by six or we multiply it by ten minutes, because
division by/ six takes a sixth of the dividend, as 2
we take a sixth of it by multiplication by a sixth,/
and it is, of sixty[3] minutes, ten minutes. So we 3
multiply half of the digits/ by seven, then by ten 4
minutes; the result will be feet of that shadow of
a seven(-foot gnomon).

If we seek to dispense with the halving in 5
this operation, we multiply/ the [given][4] digits 6
by seven, and there results the double of what resulted first.

So it is necessary that we divide it by twice 7
what we were dividing it by at first, so that/ the 8
division will correspond in the two. And the double
of the divisor is twelve. It leads to what preceded,/
whether we divide by it or we multiply the dividend 9
by five minutes, and there comes out/ by both 10
(methods) this shadow in feet.

Some of the computers use for simplification 11
five times the two numbers,/ and so they multiply 12
by five, times the seven, which is thirty-five, and
they divide by five/ times the twelve, which is 13
sixty, because the ratio of any number to another
is/ the ratio of the same multiples of them, and 14
the sixty is the denominator of the fractions in

[1] Text نقصنا ; read نصفنا .
[2] Text مخطاة ; read معطاة as in the MS.
[3] Text ستين ; MS الستين .
[4] Text مخطاة ; read معطاة as in the MS.

(this) craft. And/ hence it is necessary to mul- 46:15
tiply the digits by thirty-five minutes, and
there result/ the seven-fold feet. 16

This is the simplification of not using the
two integer numbers, since two fifths of them/ func- 17
tion as the two, and thus the use of multiples is
better.

If we want to transform the two digits of the 18
example into fractional feet, we multiply the two
of them (the digits) by/ six and a half, and we di- 19
vide the thirteen by twelve, and there results a
foot plus a half of a/ sixth, and those are the 47:1
feet of this shadow, and by advancing the division,
the division will be/ thirteen (parts) of twenty- 2
four. These two numbers are the [doubles][1] of the
divisions of the two gnomons,/ or we double their 3
divisions, and when we multiply the digits by thir-
teen and divide the result by/ twenty-four there 4
result the feet of the fractional shadow. The six
and a half equals/ half of twelve plus half of a 5
sixth of it. And so if we increase half the digits
of/ the shadow by half of its sixth there result the 6
fractional feet. But the increase of half/ the sixth 7
will be by multiplication by thirteen and division
of the result by twelve.

If we want the halving of the digits the 8
matter reduces to multiplying it by thirteen/ and 9
dividing what results by twenty-four, and that has
been presented before.

Abū Ma•shar in the fifty-seventh chapter of 10
his zīj multiplies the shadow (in)/ digits by six 11
feet and a half, and he divides the result by twelve
digits,/ so that there results for him the shadow in 12
fractional feet. Then he puts in the table for the
parts of an eighth of/ a circumference, six feet and 13
two thirds of a foot, contrary to his own computation,
and indeed we mentioned the situation as to/ those 14
who lifted (material) from his zīj. These are all
of the six couplings.

If the [given][2] shadow is in the sevenfold 15

[1] Text انصاف ; read اضعاف .
[2] Text الغطي ; read المعطي .

feet and we want it in parts, we multiply it/ by 47:16
sixty and divide the result by seven, and the
parts come out.

If we want to convert the sevenfold feet 17
into digits we multiply them by/ twelve and divide 18
the result by seven, or we halve the feet and in-
crease its half by its (the half's)/ sixth by mul- 19
tiplying (the half)// by seven and dividing by f.203a
six, or by multiplying by seven,/ then by ten 48:1
minutes. Or if we wish, we double the number of
feet, and we subtract from its double/ its seventh 2
by multiplying by six and dividing by seven.

Similarly, if we subtract from the feet of 3
the [given][1] shadow a seventh of them and double/
the remainder (to obtain the result), or, like what 4
preceded, we can multiply the feet by sixty and di-
vide the result/ by thirty-five, there resulting for 5
all of these (operations) the desired digits.

The reverse of the preceding simplification 6
is that we divide the feet by/ thirty-five minutes, 7
but the [advantage][2] of simplification in multipli-
cation is not present in/ division. 8

If we want to convert the sevenfold feet into 9
fractional feet/ we multiply them by six and a half 10
and we divide what results by seven to obtain the
desired (thing)./ And if we want to, we subtract 11
from the sevenfold feet half a seventh of them by
multiplying by/ thirteen and dividing the result by 12
fourteen. And verily, out of the couplings,/ nine 13
are complete.

If the [given][1] shadow is in fractional feet 14
and we want to convert it into parts we/ multiply 15
them by a hundred and twenty and we divide the result
by thirteen to obtain/ the parts. The choice is up 16
to you in these two numbers; if you want, make them/
halves of divisions of each of the two gnomons, or 17
if you want, make them double/ the two gnomons. The 18
matter in practice is the same, and the objective at
the end is the same.

[1] Text الغطي; read المعطي .
[2] Text مزوة ; read مرية .

However, as for the conversion of these 48:19
fractional feet into digits, we multiply them by/
twenty-four and divide the results by thirteen, 49:1
and these two are halves of the divisions of/ the 2
two gnomons, which Abū Maʿshar used in the (above-)
mentioned chapter of his zīj, as they are,/ without 3
reduction into halves.

As for their conversion into the sevenfold 4
feet, it is that we multiply by/ fourteen and divide 5
what results by thirteen. Thus all of the/ twelve 6
couplings are complete.

THE NINTH (CHAPTER) ON THE DIRECT SHADOW 49:7

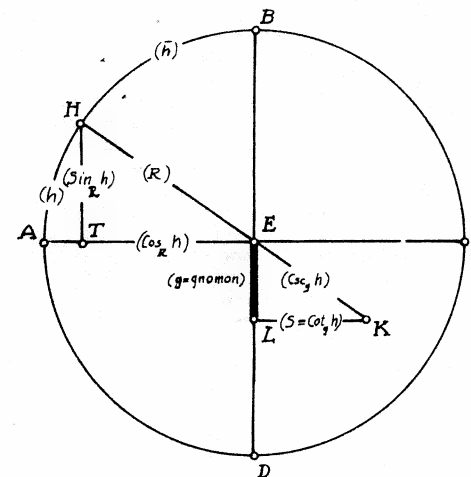
(THE COTANGENT) AND THE ALTITUDE, AND THE EXTRACTION/ 8

OF ONE OF THE TWO FROM THE OTHER IF (EITHER IS) UNKNOWN

The ratio of the gnomon to the hypotenuse of 9
the shadow (*or* cosecant) is as the ratio of the sine
of the altitude to/ the total sine. 10

Let *ABG* (Figure 8) be the circle of altitude
with center *E*,/ representing[1] the gnomon head, and 11
AEG the common part between the/ plane of the horizon 12
and the plane of this circle, and *B* (and) *D* are the

Figure 8

[1]Text القائم ; MS القيام.

two poles of the horizon./ We lay off *EL* equal to 49:13
the gnomon, and the sun is at point *H*. So/ *AH* will 14
be its altitude and the perpendicular *HT* is the
sine of this altitude, and *HB* the/ complement of 15
its altitude, and *ET* is equal to its sine. We ex-
tend ray *HEK*/ and *LK* perpendicular to *HL*. So *LK* will 16
be the cotangent/ of the altitude *AH*, and *KE* will be 17
the cosecant, and by virtue of the parallelism of
the two lines *LK* (and)/ *TE* the angle *HET* will be the 18
external (one) equal to angle *EKL*,/ and the two 19
angles *T* (and) *L* will be right angles. So the trian-
gles *EKL* (and) *HET* will be similar,/ and the ratio 50:1
of *EL*, the gnomon, to *KE*, the cosecant, will be as
the ratio of *HT*,/ the sine of the altitude, to *EH*, 2
the total sine.

If we are given the shadow at a certain time, 3
and we want to find the altitude/ of the sun for 4
that time we multiply the shadow by its equal and
the gnomon by its equal and we take [the square
root][1]/ of the sum, and it will be the cosecant. 5
Then we divide by it the product of the gnomon by the
total/ sine, and there comes out the sine of the 6
altitude. We find its corresponding arc in the sine
table and there comes out the/ altitude of the sun 7
at the time of that shadow. Thus we operate for the
sine of any/ named arc if it is given. And because 8
the gnomon and the total sine are fixed in any
zīj// at/ certain amounts, it is possible to f.203b
assume as a base in the operation the product of 9
one of the two multiplied by the other,/ to be used 10
always, as is the case in some of the zījes. If
this is expressed in parts/ and in the amount of the 11
total sine, the product of the sixty parts of the
gnomon by the sine of/ Ptolemy will be 3600, and by 12
the Indian sine 1[5]0[2].

The product of the digits of/ the gnomon by 51:1
the sine of Ptolemy is 720, and by the Indian sine
3[0][3]./ The product of the feet of the seven-foot 2

[1]Missing in text and MS.
[2]Text ١٠٠ ; MS ١٥٠.
[3]Text ٣٥ ; MS ٣٠.

gnomon by [the sine of][1] Ptolemy is 420, and by 51:2
the/ Indian sine (it) is seventeen parts and a 3
half, which is half of/ 35. The product of the 4
fractional feet of the gnomon by Ptolemy's sine
is 390,/ and by the Indian sine sixteen parts and 5
a quarter, which is a quarter of 65./ So if the 6
gnomon is made two parts and a half to be equal to
the Indian sine the product/ of these parts times 7
the Ptolemaic sine would be 150, and for their
sine it would be six and a quarter,/ and in quar- 8
ters it is 25.

When one puts together the operations of the 9
workers in this field about that,/ the routes they 10
travelled and the numbers they put will not remain
hidden. Such is the case with Muḥammad b. Ibrāhim/
al-Fazārī, and Ya•qūb b. Ṭāriq, and Muḥammad b. 11
Mūsā al-Khwārizmī, and Ḥabash/ al-Ḥāsib, and Abū 12
Ma•shar al-Balkhī, and al-Faḍl b. Nadīm al-Nayrīzī,
and Muḥammad b. Jābir/ al-Battānī, and Abū al-Wafā' 13
al-Būzjānī. All of these explained in their zījes/
that if the shadow (cotangent) is squared and the 14
gnomon is squared and the (square) root of their
sum is taken, it/ will be the cosecant (hypotenuse 15
of the shadow), because *KE* in the preceding figure
is the hypotenuse of the right triangle with legs
KL (and)/ *LE*. Then some of them mentioned the 16
squaring of the gnomon absolutely (i.e. without
regard to units), while others/ specify the number 17
(i.e. unit) of its square according to the number
assumed for the gnomon in their (own) zīj, as a
hundred/ and forty-four [for the digits, and forty- 18
two][2] and a quarter for one of the two kinds of
feet and forty-nine for/ the other kind, and three 19
thousand and six hundred for the parts. Such is
the situation in the Shāh Zīj/ for the digits. 52:1
When the cosecant is determined for all of them,
some of them proceed to the/ cosine of the altitude, 2
and some to the sine of the altitude itself.

As for those who proceed toward the cosine 3
of the altitude, they multiply the/ assumed cotan- 4
gent by the total sine and divide the result by

[1]Text في بطلميوس ; MS في جيب بطلميوس.
[2]Missing in the text.

the cosecant so that there comes out for them/ 52:4
the cosine of the altitude, because the ratio of 5
LK to *KE* is as the ratio of *TE*/ to *EH*, and *ET* is 6
equal to the sine of arc [*B*]*H*[1], the complement of
AH,/ the altitude. 7

These are al-Khwārizmī, in one of his operations, and al-Nayrīzī, and al-Battānī,/ and likewise 8
Kūshyār in his Jāmiʿ Zīj. But the total sine, which/
for him is graduated in sixty parts, makes him replace multiplication by depressing (the cosecant) 9
one place. So, he said divide/ the cotangent by 10
its cosecant depressed, that is, multiplied by
sixty. And there comes out the cosine of the/ altitude. 11

However, those who proceed toward the sine 12
of the altitude itself, they divide by the/ cosecant 13
the product of the gnomon and the total sine, because the ratio of *EL* to/ *EK* is as the ratio of *HT* 14
to *HE*. But neither the/ gnomon nor the total sine, 15
as we said, is of interest as to its (absolute)
magnitude. And so they took the product of/ one 16
of the two by the other, as demanded by the zīj.
However al-Fazārī and al-Khwārizmī/ and Yaʿqūb b. 17
Ṭāriq, and Abū Maʿshar and the author of the Shāh
Zīj, prescribe the/ division of a thousand and 18
eight hundred by the cosecant, and it is the product of a hundred and fifty/ by twelve. 19

It is necessary to attach to this number 53:1
the mention of the minutes, guarding from the/
error of one who imitates but does not understand. 2

However, Ḥabash and al-Battānī prescribe 3
the division of seven hundred and twenty by/ the 4
cosecant, it being the product of sixty and twelve,
and by these operations/ the altitude becomes known 5
to them.

For the inverse of this, if the altitude is 6
assumed known and the shadow (cotangent) of the
gnomon is wanted/ for that time:// The ratio f.204a
of *HT*, the sine of the altitude in the preceding
figure, to/ *TE*, its cosine, is as the ratio of *EL*, 8
the gnomon, to *LK*, its shadow, and from this the gnomon

[1] Text طح ; read ح.

is/ multiplied by the cosine of the altitude and 53:9
the result is divided by the sine of/ the altitude, 10
and the shadow results.

This operation in the Shāh Zīj, and (the 11
zījes) of Yaʿqūb, al-Khwārizmī, Ḥabash,/ Abū Maʿshar, 12
al-Nayrīzī, and al-Battānī does not differ except (to
the extent) that the above operation differs./ I 13
mean that some of them omit mentioning (the units of)
the gnomon when it is multiplied by itself, while
others specify/ its parts according to what has been 14
assumed in their zījes.

As for al-Nayrīzī, he multiplies by the total sine instead of multiplying by/ the gnomon, be- 15
cause both of them, according to him are sixty parts. 16
And what/ Kūshyār prescribes about dividing the co- 17
sine of the altitude by the sine of the altitude,
depressed, is/ exactly what (al-Nayrīzī) prescribes. 18
The depression (operation) is to (divide) the multiplicand (sic) by the sine of the altitude/ dep- 19
ressed by sixty, which is the (number of) parts of
the gnomon according to him. And Abū al-Wāfa'/
prescribed like him, except that he did not depress 54:1
it, since he had assumed the gnomon to be one.

The reader of the book of Abū Saʿīd Aḥmad b. 2
Muḥammad/ ʿAbd al-Jalīl al-Sijzī, "On Operation(s) 3
with the Astrolabe" (*Fī'l ʿamal bi'l-asṭurlāb*), may
think that the form of the shadow in it is different/ from what has preceded because he follows in 4
it the method of transformation, which we know from/
the preceding picture, however the computation is 5
that which preceded.

As for the proof, he cleverly took *HT*, in the 6
two first quantities/ of the four in the proportion, 7
as the sine of the altitude, and *TE* as its/ cosine. 8
Then he took *HT*, in the latter pair of magnitudes,
as the gnomon and/ *TE* as its shadow, so that the 9
ratio of *HT* to *TE* in some unit comes to be equal to
the ratio of/ *HT* to *TE*, in some other unit, and that 10
is why we likened it to the transformation.

We say that the ratio of *HT* to *HE* is as the 11
ratio of *EL*/ to *EK*. And if we divide the product of
the gnomon times the total sine by the sine of/ the 12

altitude, there results the cosecant. It is the 54:13
hypotenuse whose legs are in (the shadow) and the
gnomon./ And if we subtract from the cosecant 14
squared the square of the gnomon,/ and take the root 15
of the remainder it will be the shadow at that al-
titude, but the gnomon is invariant in/ amount, even 16
though the number of its divisions varies, so that
its product by itself (its square) will be invariant
[except][1]/ according to their (the divisions') va- 17
riation. This was followed by al-Fazārī and Ḥabash;
one of the two divided by the/ sine of the altitude 18
a thousand and eight hundred, since the total sine
according to him is [a hundred and fifty; the other
by seven hundred and twenty, since the total sine
according to him is][2] sixty./ And so there comes 19
out the cosecant for both of them. Then we sub-
tract from its square a hundred and forty-four,/ 55:1
which is the square of the gnomon, and there remains
to them the square of the shadow.

However, Abū al-Wafā' divided the total sine 2
by the sine of the altitude,/ and there came out for 3
him the cosecant, because when he assumed the gnomon
to be one, the product of/ the total sine by it is 4
it itself, exactly. And so, his dividing it by the
sine of the altitude has the same effect/ as dividing 5
the product of the total sine and the gnomon. When
he obtained the cosecant/ he operated on one of its 6
two legs as (in) what preceded in taking the (square)
root of the difference between the squares of/ the 7
cosecant and the gnomon.

In the other method the cosecant is multiplied 8
by the cosine of the altitude,/ because the ratio 9
of *EK* to *KL* is as the ratio of *EH* to *ET*. And so if
KE is/ multiplied by *ET*, the division by *EH* cannot 10
be dispensed with unless/ we make it also one, which 11
we did not do and thus it should be performed in or-
der to make it come out right.

It has been found in various anonymous opera- 12
tions that if nine hundred and seventy-five is divi-
ded by the sine of/ the altitude, then square what 13

---

[1] Text الى ; read الا .
[2] Missing in the text.

results/ and subtract from it forty-two and a 55:14
quarter, the (square) root of what remains is the
cotangent, and that is exactly/ what was explained 15
before.

As for its originator, he attempted to find 16
the shadow (*or* cotangent) in fractional feet, and
so he divided/ the product of the total sine, taking 17
it as a hundred and fifty, by the gnomon in those
feet,/ divided by the sine of the altitude to obtain 18
the cosecant as was explained previously. However,
as for the/ forty-two and a quarter, it is the 19
square of this gnomon,// which is a [quarter][1]/ f.204b
of a hundred and sixty-nine. However, by the 56:1
seven-foot gnomon it would be/ forty-nine. 2

If we want the shadow according to Ptolemy's 3
method in such a situation, (we do)/ as he explains 4
it in the fifth chapter of the second book of the
Almagest, because the angle/ *KEL* in the preceding 5
figure (Figure 8) is equal to the complement of the
altitude, and so angle *EKL*/ equals the altitude, 6
and that is in units such that in them four right
angles are/ three hundred and sixty parts. 7

So, in such units that two right angles 8
make three hundred and sixty parts,/ angle *KEL* (is) 9
double the complement of the altitude, and angle
EKL (is) double/ the altitude. And so *EL* is the 10
chord of double the altitude, and *LK* the chord of
double the complement of/ the altitude in the cir- 11
cle circumscribing triangle *EKL*. And so this tri-
angle is known as to/ sides in units such that in 12
them *EK* is a hundred and twenty parts. But the
gnomon/ *EL* is assumed in amount, and its ratio to 13
shadow *LK* is as the ratio of *EL*,/ read as the chord 14
of double the altitude, [to *LK*, read as the chord
of double its complement][2]. And so shadow *LK*,
hence, is known in the scale of the gnomon/ *EL*. 15
This is Ptolemy's method.

Because halves of chords have the same ratio 16
as their doubles, so if we halve the chords/ (above-) 17
mentioned, their arcs will no longer pertain to the

---

[1] Text بالازيد ; read بالاربع as in the MS.
[2] Missing in the text.

doubles and they will become their sines, and/ the problem is reduced to the first method about which we spoke concerning the Shāh Zīj and a group/ of authors of zījes, and does not differ in anything (insofar) as the requirements of computation (are concerned), although/ Abū al-Ḥasan al-Ahwāzī thought it was different, thinking that it was a method other than that (known) to the people. 56:17 18 19 57:1

THE TENTH (CHAPTER) 57:2

ON THE REVERSED SHADOW (THE TANGENT FUNCTION) AND THE ALTITUDE AND THE EXTRACTION OF ONE OF THE TWO FROM THE OTHER IF IT IS UNKNOWN 3

For the tangent function let us repeat the preceding figure and lay off the gnomon from/ diameter (sic) *EG* (Figure 9). *LK* will be the reversed shadow of the gnomon *EL*. And the two triangles/ *HTE* (and) *KLE* will remain similar, so the ratio of *LK* 4 5 6

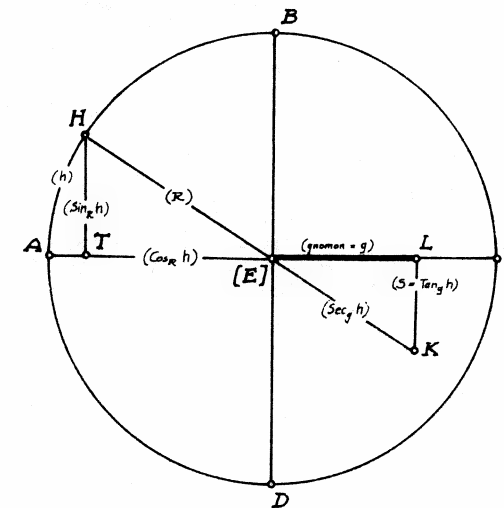



Figure 9

to *KE* / is as the ratio of *HT* to *HE*, and the 57:7
ratio of *HT* to *TE*/ is as the ratio of *LK* to *LE*. 8
If we are given that the tangent is known/ and 9
we need its altitude, we would take the root of
the sum of the squares of the/ tangent and its 10
gnomon so that there would result the secant (*or*
hypotenuse of the reversed shadow). Then we multiply/ the tangent by the total sine and we divide 11
the result by the secant,/ and there will result 12
the sine of the altitude.

To this Kūshyār referred in his saying divide 13
the tangent by its secant,/ depressed, that 14
is multiplied by the total sine which, according
to him, is sixty parts. If/ the [given][1] were the 15
known altitude and we desired its tangent, we multiply the sine of the/ altitude by the gnomon and 16
we divide the result by the cosine of the altitude,
and there would result/ its tangent in divisions 17
of its gnomon, and al-Battānī took it in exactly
the same manner.

However, Kūshyār prescribed that the sine 18
of the altitude be divided by its cosine,/ depressed, 19
so that the tangent would come out, and the depressing of the sine of the altitude is (like) multiplying it by/ the total sine which, according to 58:1
him, equals the gnomon.

As for Abū al-Wafā' he eliminated the 2
(operation of) [depressing][2] from the operation
because he supposed the gnomon to be/ one, or in 3
another place he prescribed the division of the
total sine by the cosine of the altitude/ so that 4
there came out for him the secant, and that is because the ratio of *HE* to/ *ET* is as the ratio of *KE* 5
to *LE*. And if the product of *HE* by/ *LE* is divided 6
by *ET* there results *KE*, but *LE* according to
him is one./ So the product of *HE* by *LE* will be 7
exactly *HE*. If/ the cosecant is obtained, extraction 8
from it of the cotangent is by the two methods

---

[1] Text المغطى ; read المعطى. In text Figure 9, for > at center read ه ; the ل is misplaced.

[2] Text الخط ; read الحط as in the MS.

presented in our explanation of the method/ for 58:9
this (kind of) shadow (i.e. the cotangent), exactly.
This shadow, in addition to its utility in// f.205a
computations of astronomical arcs/ is useful 10
(also) in operations with hours by instruments
which are raised up, like the ***mukḥula*** (collyrium
container) and the ***sauṭ*** (whip),/ and suchlike. 11
Sometimes it is useful in observations, such as
(that of) the summer solstice, but/ its time is 12
difficult to obtain, it being simpler, easier,
and/ more exact by (using) the reversed shadow, 59:1
while the time of the winter solstice (is easily
obtained) by the direct shadow, and not/ the 2
reversed shadow.

THE ELEVENTH (CHAPTER) 59:3

ON THE (QUALITIES) COMMON BETWEEN THE 4
TWO TYPES OF SHADOW (TANGENT AND COTANGENT),
AND THEIR RELATIONS,
AND THE EXTRACTION OF ONE FROM THE OTHER 5

The very same shadow will be a cotangent of 6
one arc and the tangent of its complement./ That 7
is that line *AEG* (in Figure 10), if it were in the
surface of the horizon,/ the zenith would be point *B* 8
and the altitude *AH*, and *LK* would be the cotangent
of the gnomon *EL*./ But if one computes with point *A* 9
as the zenith and with line *BED* in the surface of/
the horizon, the altitude would be *BH*, and the
gnomon *EL* parallel to the horizon,/ and [*L*]*K*[1] would 10
be the tangent of the altitude *BH*, and *LK*/ the co- 11
tangent of arc *AH* and the tangent for arc *BH*, and

Figure 10

[1]Text ا ; read ل. In the second text Figure 10, ر and ل are missing.

the gnomon (would be)/ the mean (proportional) 59:13
in the ratio between the two shadows of the arc,
the one direct the other reversed (i.e. the one
the cotangent the other the tangent).

So, reverting to the figure, *KL* is the co- 14
tangent of arc *AH*, and we [set up][1]/ upon *E* dia- 15
meter *TEM* perpendicular to *HK*. *KL* will pass
through/ *M*. And if we imagine that *BED* is in the 16
surface of the horizon and *G* is the zenith/ so 17
that the altitude will be *BT*, *LM* will be its
tangent, but/ arc *BT* is equal to arc *AH*, and 18
verily it (*KL*) will be its (*BT*'s) cotangent./
Because angle *KEM* is a right (angle), and thus 19
is in the semicircle whose diameter is *KLM*,/
the triangles *MEK*, *MEL*, (and) *KEL* will be simi- 60:1
lar, and hence the ratio of/ *KL*, the cotangent, 2
to *LE*, the gnomon, will be as the ratio of *LE*,
the gnomon,/ to *LM*, the tangent. And so the square 3
of the gnomon will be equal to the product of/ the 4
cotangent and the tangent of the same arc.

For purposes of simplicity we can lay off 5
EZ equal to the gnomon *EL*, and we erect/ at *Z* 6
[perpendicular][2] *SZO* to *AEG*, making the two tri-
angles *KEM*/ (and) *SE*[*O*][3] similar. *ZO* will be 7
the tangent of arc/ *AH*; it will be equal to *LM*, 8
the tangent of arc *BT*, which is equal to/ arc *AH*. 9
And so the gnomon *EL* will then be the mean pro-
portional between the two shadows, *LK*,/ the di- 10
rect, and *ZO*, the reversed (i.e. the cotangent
and the tangent), and hence if one of the two
kinds of shadows is known/ to us for an assumed 11
arc, it is possible for us to ascertain from it
the other by dividing by the known one/ of the two, 61:1
whether the cotangent or the tangent, the product
of its gnomon by itself,/ and extracting the 2
other, the unknown, in units of the divisions
of that gnomon.

[1]Text: نجبن ; read نجيز.
[2]Text: عمودى ; read عموداً as in the MS.
[3]Text: ح ; read ع.

THE TWELFTH (CHAPTER) 61:3

ON TABLES/ CONTAINING SHADOWS, EXCLUSIVE OF 4

THEIR COMPUTATION, AND HOW TO OBTAIN THEM

(THE FUNCTIONS)

It is customary among authors of zījes to put 5
the values of/ the shadows corresponding to their 6
arcs in tables arranged part by part (degree by de-
gree), and this arrangement is befitting/ it, and 7
this is how we put them.

So we enter with the arc of the (given) al- 8
titude in the column of the independent variable,
considering the first (column) value if we want/ the 9
cotangent, and the second if we want the tangent, to
find the shadow opposite that/ altitude in the table 10
whose value we are seeking. In more complicated/
zījes there will be one column (of the independent 11
variable). If we want one of the two kinds (from
the table of the other), the cotangent from the/ tan-
gent table, or the tangent from the cotangent table, 12
we subtract the altitude,/ I mean the assumed arc, 13
from ninety, and we enter with the remainder in the
column of the independent variable,/ and we take 14
what is opposite it and it will be the desired
(thing).

Thus if we use in computation the complement 15
of the altitude, (and not the altitude itself) there
would come out for us a shadow/ of a kind different 16
from that for which that operation (i.e. table) was
intended. And when the shadow is assumed and one
wants/ its arc, that shadow is sought in its table, 17
and its arc will be opposite it/ in one of the two 18
columns of the independent variable, if it were the
cotangent, in the first of the two, and if/ the tan- 19
gent, it would be in the second of the two. This is
determined also from the heading above the column

as/ one determines the type of the shadow from 62:1
the heading above its table.// f.205b

The Direct Shadow	The Reversed Shadow	Digits	Feet 6;30	Feet 7	Parts	The Direct Shadow	The Reversed Shadow	Digits	Feet 6;30	Feet 7	Parts	The Direct Shadow	The Reversed Shadow	Digits	Feet 6;30	Feet 7	Parts
1	89	(19) 687;23	(21) 372;21	(1) 401;0	(34) 3,437;22	31	59	(58) 19;18	10;49	11;39	99;51	61	29	6;39	3;36	3;53	(16) 33;56
2	88	343;38	(184;8) 185;45	(27) 200;28	1,718;11	32	58	(18) 19;13	10;24	11;12	96;1	62	28	(23) 6;24	3;27	3;43	31;54
3	87	(58) 228;46	(124;2) 123;32	133;34	1,144;52	33	57	(29) 18;28	10;1	10;47	92;24	63	27	6;7	3;19	3;34	30;54
4	86	171;36	(37) 92;42	(6) 100;7	858;2	34	56	17;47	9;38	(13) 10;22	88;57	64	26	5;51	3;10	3;25	(14) 29;—
5	85	(137;10) 138;25	(18) 74;16	(1) 80;0	(48) 686;43	35	55	(8) 17;6	9;17	10;0	85;41	65	25	5;36	3;2	3;16	27;19
6	84	114;10	(81) 61;20	63;36	570;52	36	54	(31) 16;7	8;57	(38) 9;38	82;35	66	24	5;21	2;54	3;7	(43) 26;—
7	83	(44) 97;46	52;58	(1) 57;2	448;40	37	53	15;55	(38) 8;37	(17) 9;—	79;37	67	23	5;7	2;46	(58) 2;39	25;53
8	82	(23) 85;22	46;15	49;48	426;15	38	52	(22) 15;21	(19) 8;20	8;58	(48) 76;44	68	22	4;51	2;38	2;50	(24) 23;14
9	81	(46) 75;45	(2) 41;3	44;12	(50) 378;49	39	51	14;48	8;2	8;39	74;6	69	21	4;36	(30) 2;29	2;41	23;2
10	80	68;23	36;52	39;42	340;17	40	50	14;18	7;45	8;21	71;30	70	20	4;22	(22) 2;25	2;33	(50) 21;—
11	79	61;44	33;26	36;1	308;40	41	49	13;38	7;26	8;3	69;1	71	19	4;9	(14) 2;15	2;25	20;40
12	78	56;27	30;35	32;56	(17) 282;57	42	48	(10) 13;18	7;14	7;46	66;38	72	18	3;54	(7) 2;2	2;16	19;30
13	77	(59) 51;18	28;9	30;19	259;54	43	47	12;52	6;58	7;30	64;21	73	17	3;40	1;59	2;8	18;21
14	76	48;9	26;4	28;5	240;39	44	46	12;25	6;44	7;15	62;8	74	16	3;26	1;51	2;0	17;12
15	75	44;47	(16) 24;15	26;7	223;55	45	45	12;0	6;30	7;0	60;0	75	15	3;13	1;45	1;53	16;5
16	74	41;51	22;40	24;25	209;15	46	44	11;35	(17) 6;16	(44) 6;45	57;56	76	14	(3;0) 2;59	(37) 1;38	1;45	14;58
17	73	39;53	21;16	(24) 22;14	196;15	47	43	11;11	(4) 6;3	6;31	55;57	77	13	2;46	1;30	1;37	13;51
18	72	36;15	20;1	21;13	184;40	48	42	10;48	5;51	6;18	54;1	78	12	2;33	1;23	1;29	12;45
19	71	(51) 34;44	18;53	20;20	(174) 176;15	49	41	10;25	5;39	6;5	52;9	79	11	(20) 2;19	1;16	1;22	11;40
20	70	32;2	17;52	19;14	164;56	50	40	10;3	5;27	5;52	50;21	80	10	(7) 2;6	1;9	1;14	(35) 10;36
21	69	31;13	16;56	18;14	156;18	51	39	9;43	5;16	5;40	48;35	81	9	1;54	1;1	(7) 1;6	9;30
22	68	(42) 29;45	16;6	17;20	148;30	52	38	9;22	5;5	5;28	46;53	82	8	1;41	0;55	0;59	8;26
23	67	28;16	15;18	16;29	141;21	53	37	9;2	(54) 4;59	5;16	45;13	83	7	1;28	(48) 0;47	(52) 0;51	7;22
24	66	(57) 26;15	14;36	15;43	134;46	54	36	8;44	4;43	5;5	43;36	84	6	(16) 1;15	0;41	0;44	6;18
25	65	25;44	13;56	15;1	(40) 128;47	55	35	8;24	4;33	4;54	42;1	85	5	(3) 1;2	(34) 0;35	0;37	5;15
26	64	24;36	(20) 13;19	14;21	123;1	56	34	(6) 8;5	4;24	(43) 4;44	40;28	86	4	(50) 0;54	0;27	0;29	4;12
27	63	23;33	12;45	13;44	117;45	57	33	(48) 7;47	4;19	4;33	(58) 38;18	87	3	0;37	0;20	0;22	3;9
28	62	22;34	(18) 12;14	13;10	112;51	58	32	(30) 7;29	4;4	4;23	37;30	88	2	0;25	0;14	0;15	2;6
29	61	21;48	11;44	12;38	108;15	59	31	7;12	3;55	4;13	36;3	89	1	(13) 0;12	0;7	(7) 0;8	1;3
30	60	20;47	(16) 11;15	12;7	(53) 103;15	60	30	6;53	3;46	(2) 4;3	34;38	90	0	0;0	0;0	0;0	0;0

(In the MS, f.205b is the table of cotangents and tangents transcribed above. It does not appear in the printed edition.)

We put after the tables that which is 62:2
most useful to know in connection with any f.206a
table./ And we say that it is (well-)known 3
that if the tables have equal (tabular) dif-
ferences/ in the (entries) opposite the column 4
of the independent variable, then the correction
for the fractions of the excess/ over the integer 5
in the column of the independent variable is
found by means of the tabular difference, (the
correction being) definite and/ completely exact. 6
If the tabular differences are unequal, then the
correction for fractions in it/ by tabular dif- 7
ferences will be approximate and not exact. The
greater the difference in the tabular/ differences, 8
the less exact it is and/ the more in error, be- 9
cause the variation in the dependent variable due
to fractions (of the argument) depends on the
variation in it due to/ the integer parts. The 10
shadow (functions) behave like this. The cotan-
gent (exhibits this) at the beginning of the al-
titude, because/ its greatest part is at sunrise 11
and sunset; however the tangent (does so) at the
maximum/ altitude because its greatest part is 12
at the sun's approach to the zenith.

Thus Kūshyār arranged the tangent table, 13
in his Jāmiʿ Zīj,/ up to an eighth of a revolution. 14
He said that for arcs exceeding forty-five degrees/
there is no way of finding the tangent except 15
by main force. However, in fact the tabular
differences for the shadow functions are so
great/ that the shadow computed (by interpola- 16
tion) can hardly be correct. But this need not
be the case if one devises an expedient for it
as did/ Ptolemy by taking two (successive) 17
amounts by which he formed segments of the epi-
cycle/ and the eccentric for (computing) the 18
equation, taking one of them three degrees/ and
the other six degrees. 63:1

However, as for the tabular differences 2
corresponding to (fractional) parts of the arc,
(they are different). This is obvious/ on 3
account of their difference for integer parts,

but it will become more obvious if with center 63:3
at/ *A* (Figure 11), the head of the gnomon, and at 4
distance *AB*, its length, (we draw) circle *BGDEM*./
We mark off on it the arcs *BG*, *GD*, *DE* (and) *EM* 5
according to some ratio of numbers/ with equal dif- 6
ferences, whether these differences are single
parts or/ a number of them. And we extend *AGZ*, 7
ADH, (and) *AET*. *BZ* will be/ the tangent of *BG*, 8
and [*B*]*H*[1] the tangent of [*B*]*D*[2], and *BT* the tan-
gent of/ *BE*. And we join *Z* (to) *D* (to) *H*, and *S* 9
(to) *M*, and because of the equality of the angles
which/ are formed at the center *A*, the two tri- 10
angles *ABZ* (and) *ADZ* will be congruent;/ the two 11
sides [*Z*]*B*[3] and *ZD* will be equal, and the angle *ABZ*
a right angle./ And the angle *ADZ* is also right. 12
So *HZ*, its (angle *D*'s) chord, is larger than *ZD*,
I mean/ *ZB*, and so *ZB* will be less than *ZH*. 13

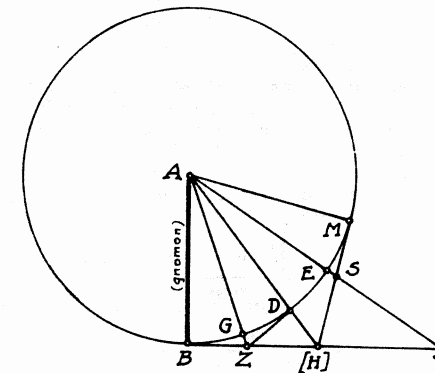



Figure 11

[1] Text: لح ; read ح as in the MS; on the figure,
for ع read ح.
[2] Text: ود ; read بد .
[3] Text: رب; read زب as in the MS.

In the same fashion *HM* will be tangent 64:1
to the circle, and because of the equality of the
two triangles/ *A[B]H*[1] and *AMH*, and the two tri- 2
angles *ABZ* (and) *AMS*, *SH*/ will be equal to *HZ*, and 3
the triangle *ASH* will be equal to the triangle *AHZ*./
And so the ratio of the triangle *AZH*, I mean *ASH*, 4
to the triangle/ *AHT*, will be as the ratio of *ZH* to 5
HT, and triangle *ASH*/ is a part of the triangle *AHT* 6
since *ZH* is a part of *HT*, (i.e. less than *HT*) and
so is/ whatever is behind *HT*. 7

This situation holds for parts of the shadow, 8
as well as for the arguments mentioned./ One should 9
be cautious in using the tangent (table) not to
exceed an arc of an octant./ If we want to multiply 10
a number by the tangent of an arc which is greater
than forty-five parts, we divide/ that number by 11
the tangent of its complement. And if we want to
divide it by the tangent of an arc, we multiply it/
by the tangent of its complement. 12

Let *A* (Figure 12) be the[2] tangent of the 13
given arc and *B* the tangent of its complement, and
(it is immaterial) whether/ we understand the state- 14
ment as referring to tangents of the same type, or
whether we associate two tangents with/ the given 15
arc, they being of different kinds. This is
because the cotangent of an arc is/ the tangent of

[E]
the division
(i.e., quotient)

[T]
number (to be) divided
by the tangent of the arc

[B] (= Tan $\bar{\theta}$ = Cot θ)
the cotangent
of the arc

[G]
the gnomon

[A] (= Tan θ)
the tangent of the arc

Figure 12

[1] Text الع ; read اع as in the MS.

[2] Letters are missing in the text; restored from the MS.

its complement. We put the gnomon *G* as the 64:16
mean proportional between *A* and *B*, as we/
explained above. And we suppose it, (to be) one 17
for simplifying the understanding in this// f.206b
respect. Indeed, we do not need/ now to deal 18
with its parts. Let us divide the number *T* by *A*,
and there will come out *E*./ And because the prod- 19
uct of *A* by *E* is *T*, and the product of *A* by *B*/
is *G*, so *A* is therefore the common altitude 65:1
between *T* (and) *G*. And its (*B*'s) ratio to *E*/ is
as the ratio of *G* to *T*. So the product of *B* and 2
T equals the product of/ *E* and *G*. But *G* is one, 3
and so its product with *E* is *E*. So the product of/ 4
B with *T* is *E*, and that is what we wanted to explain.

In like manner we show that the cotangents of 5
assumed arcs are, with their/ tangents in propor- 6
tion by equivalence. So [let][1] the two arcs *AH* and
AZ (Figure 13) be/ assumed, and *EK* the gnomon. We 7

Figure 13

[1] Text فليس ; read فليكن . On the figure *H*, *K*, and *D* are restored from the MS.

extend *HEL* and *ZEC*. And/ *KL* will be the co- 65:8
tangent of arc *AH*, and *KC* the cotangent of arc
AZ./ Then let us suppose the gnomon to be *EM*, 9
and we extend *MO* parallel/ to *EK*. It intersects 10
HL at *S* and *ZC* at *O*./ So *MS* will be the tangent 11
of arc *AH*, and *MO* the tangent of arc/*AZ*. 12

I say that the ratio of *LK* to *CK* equals the 66:1
ratio of *MO* to/ *MS*, and that is because the gnomon 2
is the mean proportional between *LK* (and) *MS*./
And the product of one of the two by the other 3
equals the square of the gnomon, and thus (also)
it is the mean proportional/ between *CK* and *MO*. 4
So the product of one of the two by the other equals
the square/ of the gnomon. And the product of *LK* 5
by *MS* is therefore equal to the product of *CK*/ by 6
MO, and so the ratio of *LK* to *CK* is, with the ratio
of *MO* to/ *MS* in proportion by equivalence. I mean 7
that the ratio of the cotangent of arc *AH* to the co-
tangent of arc/ *AZ*, is as the ratio of the tangent 8
of arc *AZ* to the tangent of arc *AH*,/ and that is 9
what we sought.

In order to find the ratio between the shad- 10
ows of the two arcs we suppose the two arcs *AH* (and)/
AZ (Figure 14) to be different, the smaller of the 11
two (being) *AH*, with sine *HT* and cosine/ [*T*]*E*[1], and 12
its tangent *MS*. The sine of the larger is *ZK*, and
its/ cosine *KE*, and its tangent *MO*. We extend *SC*/ 67:1
parallel to *EO*, and we made *ES* a mean in the pro- 2
portion between *SC* (and)/ *OE*. So the ratio of *SC* 3
to *SE* is as the ratio of the sine of the angle/ *SEM*, 4
I mean *HEA*, to the sine of angle *E*[*C*]*S*[2], whose sine
is the sine of/ angle *SCM*, which is equal to angle 5
ZEA. The ratio of *SE*/ to *EO* is as the ratio of *FE* to 6
EZ, which is equal to *EH*,/ and the ratio of *FE* to *HE*, 7
is the ratio of *EK* to *ET*./ The ratio of *SC* to *OE* is 8
therefore compounded of the ratio of *HT* to/ *ZK* times 9
the ratio of *EK* to *ET*. But the ratio of *SC* to/*OE* is 10
the ratio of *MS* to *MO*. Indeed, it is clear that the
ratio of the tangent/ of the smaller of the two arcs 11
to the tangent of the larger of the two is compounded

[1]Text كه ; read طه.
[2]Text قس ; read محص as in the MS.

Figure 14

of the ratio of the sine/ of the smaller to the 67:12
sine of the larger times the ratio of the cosine
of the larger to the cosine/ of the smaller. 13

It is evident also that it is compounded 14
of the ratio of the sine of the smaller to/ its 15
cosine times the ratio of the cosine of the larger
to its sine. That is because// the ratio of f.207a
MS/ to *ME* is as the ratio of *HT* to *TE*, and the 16
ratio of *ME*/ to *MO* is as the ratio of *EK* to *KZ*. 17
And hence the ratio of *MS* to *MO* is compounded of
the ratio of *HT* to *TE* times the ratio of/ *EK* to *KZ*. 18

However, as for the two hypotenuses of 68:1
the shadows (here secants), the ratio of *SE* to *EO*
is/ the ratio of *FE* to *EZ*, which we explained as 2
being equal to the ratio of *EK* to [*E*]*T*[1]./ And so 3
the ratio of the secant of the smaller of two arcs
to the secant of the/ larger is as the ratio of the 4

[1]Text ك ط ; read هط.

cosine of the larger to the cosine of the smaller, 68:4
and that is/ what we wanted to explain. 5

If we arrange what we mentioned in a pic- 6
ture like this (Figure 14) showing the two cotan-
gents,/ it is clear concerning the first ratio that 7
the ratio of the cotangent of the smaller arc/ to 8
the cotangent of the larger arc is compounded of
the ratio of the sine/ of the larger to the sine 9
of the smaller times the ratio of the cosine of the
smaller to the cosine of/ the larger. 10

It is clear concerning the [second][1] ratio 11
that the ratio of the (co)tangent of the smaller
to the (co)tangent/ of the larger is compounded of 12
the ratio of the cosine of the smaller to its sine
times the ratio of/ the sine of the larger to its 13
cosine.

And it appears for the two hypotenuses 69:1
(here cosecants) that the ratio of the cosecant of
the smaller of the two arcs to the/ cosecant of 2
the larger of the two equals the ratio of the sine
of the larger to the sine of the smaller, and that/
is what we were directing ourselves toward. 3

---

[1] Text التامة ; read الثانية as in the MS.

THE THIRTEENTH (CHAPTER) 69:4

ON FIXING THE KINDS OF SHADOWS

ON THE ASTROLABE, SO THAT THEY WILL BE USEFUL 5

FOR WHAT FOLLOWS

Ḥamza al-Isfahānī explained in his book 6
"Contrasts", (*al-Muwāzina*) that "astrolabe"/ is a 7
Persian expression which was Arabicised, and that
it is *a[s]tāra*[1]*yāb*, that is "the finder of the stars"./
It is possible that this name for it among the Per- 8
sians was derived from the action/ it performs, or 9
perhaps it was Arabicised from the Greek as the
Persian (word) may have been Arabicised. Indeed in
Greek its name is/*asṭrula[b]ūn*,[2] and *asṭru* is "star" 10
as is shown by "astronomy" being called by them/
asṭrūnūmiyā and "astrology" *asṭrulūjiyā*. It/ is an 11
instrument for which they have, concerning its cons- 12
truction and use, ancient books, while others have/
nothing pertaining to it, although they may have 13
something taken from them (the Greeks). The people
of the East (*ahl al-mashriq*) do not know/ the astro- 14
labe, and use nothing but the shadow instead.

(Some people) are so ignorant and fanatical 15
in siding with the Indians against the Byzantines
(*al-Rūm*) to the extent that one of them immortal-
ized,/ in a book of his, his saying that with a 16
a stick is the astrolabe made, and (also) the celes-
tial globe and the [armillary/ sphere][3], and upon 17
its shadows their kinds are based, and that the
scientists in the past used nothing,/ in all their 18

---

[1] Text اشتاره ; read استاره as in the MS.
[2] Text اسطرليون ; read اسطرلبون.
[3] Text الحنق ; read الحلق.

books, but the stick, since it yields the most 69:18
precise results and is nearest to the truth. And
therefore (he concludes) that/ the Indians deserve
to render judgments concerning the stars, and that 19
their error is slight because of the accuracy of
their applying/ the stick, to the extent that they 70:1
can extract the horoscope and its minutes up the
tens (of minutes).

But this saying resembles the talk of lu- 2
natics, or (the talk of) one who does not know/ a
single one of the nouns and verbs he used, so let 3
us [regard this][1] as a sneeze; we tell him *Gesundheit!*,
and we request for him (God's) forgiveness. 4

After this, we say that the custom was cur- 5
rent among astrolabe makers to put/ the cotangent 6
on its (the astrolabe's) back, along the circum-
ference of the quadrant opposite the altitude quad-
rant. When/ it is thus made, and if it is desired 7
to ascertain which of the kinds it is, put the
pointer,/ i.e., the pointer of the alidade, along 8
forty-five degrees of altitude. Thereupon look
along to/ what its other pointer falls upon of the 9
parts of the shadow. If it is twelve they (the
units) are digits;/ however, if it is seven, or six 10
and a half, they are feet; and if it is sixty they
are parts./ It is not customary to use anything 11
except digits. The sevenfold feet are found, but
only/ rarely. As to its being the cotangent or 12
the tangent in that type, this is ascertained by
the initial point. If the/ beginning of the shadow 13
and the arrangement of the numerals is from below
(increasing) toward the horizon line it is the co-
tangent,/ but if its beginning is at the horizon 14
line downward it is the tangent. That (is)/ be-
cause the forty-five is half a quadrant of altitude, 15
and the diameter passing through the/ midpoint of 16
the alidade [bisects][2] the right angle at that
(angle) at the center.// And because/ the alti- f.207b
tude and its complement are equal the shadow and 17

[1]Text فنبهه ; read فنهبها as in the MS.
[2]Text بنصف ; read ينصف .

the gnomon are equal, and therefore what [corres- 70:17
ponds to it on][1] the pointer of/ the alidade in 18
parts will be the magnitude of the gnomon.

To construct it, let the circle of the back 19
of the (above-)mentioned plate be circle *ABGD*/ 71:1
(Figure 15) divided into four parts by the two dia-
meters *AG* (and) *BD*, and let *BA* in it be the quadrant
of the altitude./ And so the position of the shadow 2
on it is the quadrant *GD*, the opposite quadrant,
making it possible, from the alidade running through/
pole *E*, to determine the one from the other. Extend 3
GL tangent/ to the circle at *G*. And let *H* be the 4
midpoint of quadrant *GD*, and the bisection of/ the 5
arc by construction is easy.

Figure 15

[1]Text مالوافيه ; MS ماىواڡه ; read مايوافيه .

That is (done) by describing, with the center at one of the two ends of the given arc, an arc/ with radius half the diameter of the circle, or any other distance greater than it, since what is less than it/ should be excluded in order to obtain a solution. Then we describe also with center at its other end/ and with the same distance as radius an arc in the direction of the first arc so that they intersect. Then we join/ their intersection to the center by a straight line, (and) extend it along its own length. It necessarily/ bisects that assumed arc. We extend *EHK* and divide *GK*/ in divisions of the gnomon, twelve for the digits, and six and a half, or two thirds, or seven/ whole divisions for the feet as (stated) previously, and sixty for the parts. Let *GY* be one of the/ divisions, or some of the divisions, and we join *E* (to) *Y*. And so *GZ* will be that division/ or divisions of the shadow on the astrolabe. We extend *GK*/ and we graduate in it what is (left) behind *AK* of it by the divisions of *GK*, I mean in divisions such that/ each one of them is equal to one division of the divisions of *KG*, until/ all of *GKL* will be divided equally in one unit.

71:6 7 8 9 10 11 12 13 14 15 16 17 18

If we want the tangent function, which is rare and we have not seen it/ used, we graduate the tangent line at *D*, and not (that) on *G*, such as line/ *DKO*, and we perform on it what we did with line *GKL*, until we carried over its/ divisions to arc *GD*. If the divisions of *DKO* also are carried/ to arc *DG* the parts of *DM*, *MK*, (and) *KO* would become on/ the astrolabe *DT*, *TH*, (and) *HZ*. And if we join the center to/ the divisions of these lines, the joining lines should not leave a trace, since we need/ only their intersections with quadrant *GHD*, where the trace/ is required.

19 72:1 2 3 4 5 6 7 8

The craftsmen also halve the quadrant [*BG*][1] at [*S*][2] and join/ *SCH*, and they extend it along its length. Then they divide *EC* in parts of/ the gnomon,

9 10 11

[1]Text: ار ; read ج . On the figure, *M* is restored from the MS.
[2]Text: ز ; read س .

and *C*[*H*][1] in its (the gnomon's) amounts, and they join the [center to][2] the divisions/, and so, as long as they are in *CH* they extend the lines along their lengths until/ they reach arc *D*[*G*][3], and that is done by putting the edges of the rulers alongside the points./ If they fall at arc *HD*, these lines themselves divide it at the/ desired point, and they continued increasing (in number) at *HW* so that the divisions of the shadow become very short/ on the arc near *G*, and they are unable to mark/ individual ones because of confusion, or even fives or tens of them.

72:11 12 13 14 15 16 17

It is difficult for them to fix their numerals and numbers because its lines accumulate,/ approaching each other like stitches around a sack. At this stage they leave them off. If the tangent were/ put on the quadrant *GD* that clustering (of marks) will occur at/ point *G*, contrary to its occurrence with the cotangent at the point *D*,/ and with both of them the shadow is drawn// on the arc of the quadrant (so that) there is no need for the alidade/ except for two pointers so that it can be complete, according to the old custom, or/ it can be halved and edged swordwise, as is the modern custom.

73:1 2 3 4 5 f.208a 6 7

[1]Text و ; read ح .
[2]Text المركزين ; read المركزوبين .
[3]Text ح ; read ج .

THE FOURTEENTH (CHAPTER) 73:8

ON FIXING THE LADDER SHADOW ON THE ASTROLABE 9

Since the situation of the shadows when they 10
exceed the amount of the gnomon is as we mentioned/
as to the clustering of its divisions which lead to 11
the accumulation of the constructed lines/ and the 12
impossibility of laying them out in practise (as
well as) the inscribing of the numbers opposite
them in letters, some/ of the leading modern crafts-
men in this art were kind enough to tackle (the 13
problem).

It was said in some of the books that it 74:1
was al-Khwārizmī, and that his trick for evading
that/ dilemma was to manipulate some arithmetical 2
(operations). So he combined both the shadow
(functions) on the astrolabe and/ called it the 3
ladder shadow.

To construct it, let *ABGD*(Figure 16) be the 4
back of the mother of the astrolabe, and we extend/
from the middle of quadrant [*G*]*D*[1], namely *H*, the 5
perpendicular *HZ* to the/ diameter *AEG*, and perpen- 6
dicular *HT* to diameter [*B*]*ED*[2]. There results the
square/ *ET*[*H*]*Z*[3], right angled and equilateral, and 7
we take two amounts *ZS* (and)/ *TC*, of the agreed 8
upon amount for the divisions of the shadow, and
SO (and) *CF* of the/ agreed upon (interval) for the 9
[alphabetical][4] numerals so that it can take the
letter (numerals) assigned to it. We extend/ *SY* 10
(and) *OM* parallel to *ZH* meeting *CY* (and) *FM* paral-
lel/ to *HT* in the points *Y* (and) *M*. And we connect 11
M, *Y*, (and) *H*. Then we divide both/ *ZH* (and) *TH* 12
into the divisions of the gnomon, I mean digits or

---

1 Text ج ; read خ .
2 Text ا ; read ب .
3 Text ع ; read ح as in the MS.
4 Text حملا ; read جملا as in the MS.

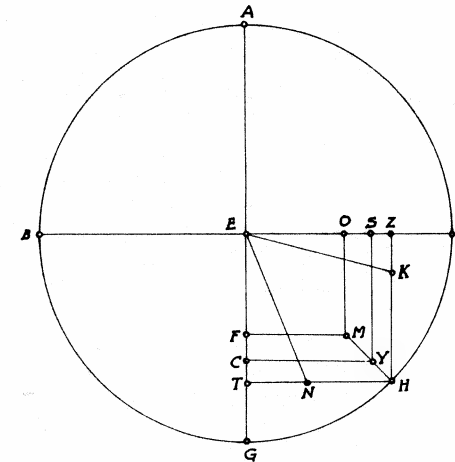



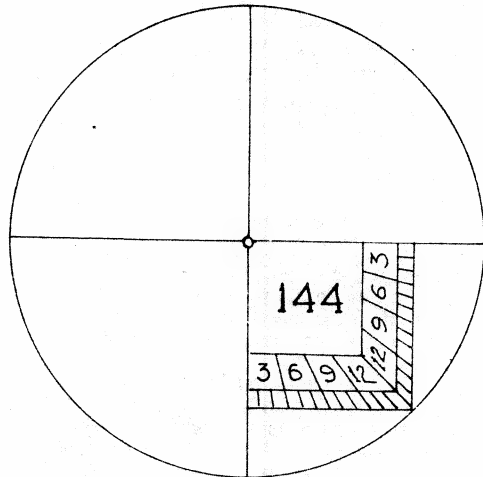
Figure 16

feet, and we join all the (points of)/ divi- 74:12
sion to *E* by lines, of which those parts between 13
the two lines of division shall be traced (per-
manently), such as the two lines/ *EK* (and) *EN* in 14
the two divisions *ZK* and *TN*.

Do not mark (permanently) between the two 75:1
lines bounding the alphabetical numerals (any grad-
uations) except those passing through/ the end 2
points of the two-digit, or three-digit,/ or four- 3
digit groups. If it is divided like this, write the
numbers beginning at *O*,/ and at *F*, until their ends 4
meet at *M*, and we inscribe between/ the two points 5
M (and) *E*, along the diagonal, the number of the
square of the gonomon in alphabetical numerals; if
in/ digits, then (it will be) one hundred and forty- 6
four, and if it were in the seven-fold feet it would
be forty-nine,/ and if it were in the fractional 7

feet it would be forty[-two][1] and a quarter. 75:7
If put as a common fraction it would be a 8
hundred and/ sixty-[nine][2] quarters. And the pic- 9
ture of this shadow on the astrolabe is complete/
thus (in Figure 17). 10
Abū al-Q[ā]sim al-Ḥasan b. Muḥammad [al- 11
Aḥwalī?][3] when constructing square *Z*[*H*]*TE*[4],/
numbers in sixth parts of the quadrant *DG*, I mean 12
fifteen(-degree intervals)/ between the points *G* 13
(and) *D*, and extends from the extremities two lines/

Figure 17

---

[1]Text اربعين ; read اثنين واربعين.
[2]Text واحد ; read وتسع.
[3]Text الاحول ; MS الاحولي.
[4]Text ه ; read ح.

parallel to the two lines *HT*(and) *HZ*, [giving][1] 75:14
him square *OMFE*.
This division is not obligatory, but it 76:1
is delegated to the approval of/ the maker and the 2
largeness or smallness of the plate. So the one
seeing it should not think that/ nothing else is 3
possible. Now (we discuss) what I have heard con-
cerning the reason for this name, and the relation
of this shadow to/ the ladder, if any. Nothing 4
comes to mind except the likening of it to the prob-
lem of the ladder written in/ the (books) "*Ḥisābāt* 5
al-muṭāraḥa" and sections of "*Al-Jabr w'al-muqābala*"
(algebra). It is a given ladder/ leaning against a 6
wall, and the distance between its base and the foot
of the ladder, or between// its head (and the f.208b
base of the wall) is known./ Then the foot of the 7
ladder is dragged along the ground by a known amount,
and the amount of displacement/ of its head along 8
the wall is desired, whether the displacement is
downward in the case of its receding from the wall,/
so that the head is depressed, or whether the dis- 9
placement is an approach to the wall so that/ the 10
head is elevated.
The similarity between them (i.e. the astro- 11
labe and the ladder) is that if the wall is *ET*(in
Figure 18) on/ the ground [*T*]*HG*[2], and the sun, for 12
example, is at point *A*, and the shadow of/ *TE* on the
ground will be *TG*. 13
If it happens that a wall *HZ* is between *H* 14
(and) *G*,/ the edge of the shadow will fall upon it 15
at *B*. And if the sun increases in altitude until it/
comes to *D*, the edge of the shadow would come to *K*, 16
as though it were the head of a ladder/ displaced 17
from *B* to *K*. If its altitude is lessened until it
is at/ *M*, the end of the shadow is displaced from 18
B to *S*.
Anyone who wants to measure the shadow *HB* 19
cannot dispense, in most/ cases, with a ladder, 77:1
and especially if the two walls are different and
lofty. When/ the shadow of *BH* is known, the shad- 2
ow *TG* becomes known also, because the ratio of/
ET to *TG* is as the ratio of *BZ* to *ZE*, and if the/ 3

---

[1]Text المحصل ; MS يحصل.
[2]Text ه ; read ط.

Figure 18

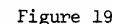
height of the shading wall, I mean *ET*, is multi- 77:4
plied by *EZ*, (which is) equal to (the distance)
between/ the walls along the ground, and we divide 5
the result by *BZ*, (which is) the difference be-
tween the/ shadow *BH* and (the height of) the wall 6
ET, there comes out the shadow *T*[*G*][1], which is that
of/ the wall upon removing the obstacle. And be- 7
cause in the astrolabe *ETHZ* was made a square,/
hence *AB*, in the computation the product of *ET* by 8
itself, is substituted for the product of *ET*/ by 9
EZ.

If one constructs this (ladder) shadow on 10
the astrolabe, he is obligated to halve/ the ali- 11
dade so that its edge, by passing through the cen-
ter, becomes a diameter. The displacement of the/
line of sight from the diameter will not affect 12
the operation in the least, since this will still
keep, for the declining of the two does not extend/
the ray passing through the two (sight-)holes away 13
from parallelism with the diameter, and the rays of
the sun are/ perceived as parallel at one location 14
because of its distance from the earth,/ and its 15
extreme height.

---
[1]Text ح ; read ج.

As to the use of this shadow and its de- 77:16
termination from the solar altitude,/ it is by ob- 17
serving the edge of the alidade passing through
the center of the astrolabe at the time we take/
the solar altitude. If it is elevated to the di- 18
visions of *TH* (in Figure 19) on the square, and
the/ desired shadow is the cotangent, the number 19
which is cut from *T* is that (which is sought).
But if it/ falls on the parts of *ZH* we divide by 78:1
what is cut off of it at *Z*, the product of the/
gnomon by itself, and that is the base placed in 2
the middle of the square, and the/ desired cotan- 3
gent comes out.

If the tangent is desired and not the co- 4
tangent, and the edge of/ the alidade falls on di- 5
visions *ZH* between *Z* and the edge, it will be it
(which is sought). But if it/ falls on the divi- 6
sions *HT* we divide the square of the gnomon by

Figure 19

what is between their intersection/ and *T*, and 78:7
the tangent results.
We explain that (by saying) that the gnomon 8
is the mean (proportional between its cotangent/
and the tangent at a single time. And so the ra- 9
tio of *KZ* to *TO* is as the ratio of/ *KZ* to *ZE* men- 10
tioned twice by repetition, and (so) the square of/
ZE equals the product of *KZ* by *TO*. And hence if we 11
divide the square of the gnomon by one of the two
shadows,/ there results the other. 12

To make it more obvious we extend *EK* (and) 13
TH along their lengths until/ they intersect at *O*. 14
The triangles *KZE* (and) *ETO* are similar, and the
ratio of/ *KZ*, which is known, to *ZE*, which equals 79:1
the gnomon, is as the ratio of *ET*,/ the gnomon, 2
to *TO*, the desired cotangent. And in (the case
of) the tangent, we extend/ *EN* (and) *ZH* along their 3
lengths until they intersect at *M*, and from the sim-
ilarity of the two triangles/ *TE[N]*[1] (and) *ZME*, 4
the ratio of *TN*, which is known, to *TE*, which is
equal to the gnomon,/ is as the ratio of *EZ*, the 5
gnomon, to// *ZM*, the desired tangent. f.209a

The knowledge of the inverse (problem) is 6
easy. If the [given][2] is the shadow, and/ the al- 7
titude is desired we look, and if the shadow is
not bigger than the gnomon/ and is direct (i.e. the 8
cotangent) we count the equal of its divisions from
point *T* toward *H*,/ but if it is reversed (i.e. the 9
tangent) from *Z* toward *H*. But if the shadow is/
greater than the gnomon, we divide the square of 10
the gnomon by the [known][2] shadow and what comes
out/ we count; if it is the cotangent (it is) from 11
Z toward *H*, but/ if it is the tangent, from *T* toward 12
H. Then we put the edge of/ the alidade, for all 13
of them, along the end, and the upper pointer of
the alidade will fall on the/ altitude of that shad- 14
ow. I have read in Abū Saʿīd Aḥmad bin Muḥammad
b./ ʿAbd al-Jalīl's book "On the Use of the Astro- 15
labe" (*Fī'l-ʿAmal b'il-asṭurlāb*) a passage about

[1] Text ر ; read ن as in the MS.
[2] Text مغطى ; read معطى as in the MS.

the ladder shadow for the/ explanation of which 79:16
we extend *KS* parallel to *HT*.

He said, "If the altitude is less than forty- 17
five degrees/ the alidade will fall along *ZH* at *K*, 18
for example, and so the (direct) shadow will be *KS*,/
the gnomon being *ES*. But *KS* is twelve, like *HT*,/ 19
and the gnomon *ES* is a part of it, but this is not 80:1
what we sought, for we want/ the opposite of that." 2

"That is that *ES* will be twelve, and the ra- 3
tio of *ES*, the known,/ to *KS*, the known, is as the 4
ratio of twelve, which represents *ES*,/ to the num- 5
ber representing *KS*, and *ES* is equal to *ZK*, and/ *KS*
is equal to *HT* and the third (element in the pro- 6
portion) representing *ES* is twelve,/ and the fourth, 7
SK, is unknown. So the second is multiplied by the
third, and it is twelve/ times twelve, which gives a 8
hundred and forty-four. And we divide that by the
first,/ which is *ZK*, and there comes out the fourth, 9
SK, and that is near (to what we sought)".

But what we explained concerning it is more 10
elegant and more illuminating by a great deal. It
is possible to project/ the divisions of the two 11
sides of the square with a ruler dividing *DH* (and)
GH so that/ the space of the square is left clear, 12
and one dispenses with making the edge of the ali-
dade like a sword (blade).

THE FIFTEENTH (CHAPTER) 81:1

ON SHADOWS MEASURED ON INCLINED PLANES OR ON OTHER (THINGS) 2

Verily, concerning the shadow and the altitude, and both of the two shadows (the cotangent and tangent) and the extraction of/ one of the two from the other, by computation and by tables, we have explained enough to suffice. And by measuring the shadow,/ the time is determined and becomes known. That is useful because, sometimes a man is not/ in a position (to utilize) immediately instruments (for determining) the altitude or the hours, and he may be afraid of missing/ a required time, while the measurement of the shadow is easy for him. So this replaces measuring/ the altitude, since (the other) is available. 3 4 5 6 7 8

So, let us now explain its construction. Ḥabash [al-Ḥāsib][1] in his [zīj][2] also has a method/ for the determination of the altitude from the shadow. It is that he measures the shadow of the gnomon;/ let it be *DE* (in Figure 20), and the point *E* is its end and *D* the base of the gnomon, and a/ perpendicular *DW* is erected upon *DE* equal to the gnomon, and *E* (and) *W* are joined. Then describe/ about center *E*, and at any distance we desire, a circle, (an arc of which is) cut off by the lines *ED* (and) *EW*/ between *B* (and) *G*, which will be the altitude of the sun corresponding to this shadow, and the validity of this is/ evident from what precedes. 9 10 11 12 13 14 15

If we drop perpendicular *GT* upon *EB* the ratio of/ *WD*, the gnomon, to *EW*, the hypotenuse of the shadow, will be as the ratio of *GT* to *GE*,/ 16 17

---

[1] Text للحساب; read الحاسب as in the MS.
[2] Text زيجيه, but MS has زيجة.

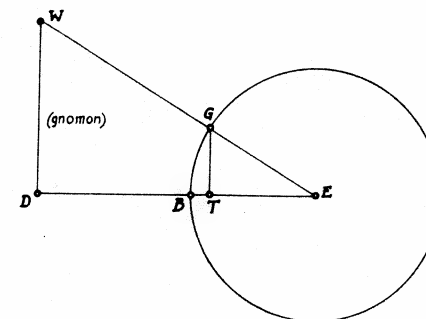



Figure 20

and *EG* is the total sine in// the circle (just) drawn. So *GT*, according to what preceded/ as to the validity of this ratio (will be) the sine of the altitude. And so arc *GB* (will be) the altitude/ of the shadow *ED*. f.209b 81:18 19 82:1

This requires a bit more explanation. As for positions,/ point *B* which is on the altitude circle can fall anywhere between the two points *E* (and) *D*,/ as well as outside, along the prolongation of *ED*, or on point *D* itself./ This is evident, and the situation is the same for all of them. 2 3 4 5

However, as for the kind of shadow (function), if *ED* is the tangent/ and we want its arc, I mean the altitude of the sun for it, we take as an example what Ḥabash mentioned so that/ there results triangle *WDE* (in Figure 21). Then we describe about center *W* and at any desirable distance/ a circle. Let (it be) *KM*, and arc *KM* of it will be the altitude of/ shadow *ED*, the tangent, because angle *WED* is equal to the altitude, and so angle/ *EWD* will be the complement of the altitude, and the cotangent of the complement of the 6 7 8 9 10 11

altitude is the/ tangent of the altitude itself. 82:12
And so the shadow *ED*, which is the cotangent of the
altitude/ *GB*, is the tangent of the altitude *KM*. 13

Figure 21

It is possible to ascertain the altitude 83:1
by measurement, and to determine the end of the shadow, *E*,/ and its position, which is *ED*. Then (suppose) the amount of the shadow is desired (measured) by a gnomon of known length./ If the case is thus, we describe about center *E* (in Figure 22) and at any distance we wish, a circle/ *GB*. And we cut off arc *BG* at *B* equal to the altitude./ We extend *EG* and drop perpendicular *GT* to *ED*, and we extend it/ along its length to *A* so that *TA* will be equal to the gnomon. Then/ we extend *AW* parallel to *ED* and *WD* parallel to *AT*. And so/ *ED* will be the shadow of the gnomon *WD* if the [altitude][1] is *GB*.

(margin line numbers: 2, 3, 4, 5, 6, 7, 8)

[1]Text الارتفاع ; MS لارتفاع.

Figure 22

This is immediately apparent from what was 84:1
previously indicated. And it is possible that/ the need for the determination of the time is so urgent 2
that it allows no time for adjusting the instrument./
(Let us suppose) the gnomon set up on a plane in- 3
clined to the plane of the horizon, (but) parallel/
to (one) standing vertically. So we mark on that 4
plane at the head of the shadow a mark/ so as to 5
retain the desired (thing), (and) we correct (it)
afterward.

That is what Yaʿqūb bin Ṭāriq mentioned of 6
its computation in his book "On the Causes..."
(*Fī'l-ʿIlal...*).

An example of that is that the plane of the 7
horizon was (taken as) *BG* (in Figure 24) and the
gnomon *AB*/, perpendicular to it, while the plane of 8
the measured shadow was *BE*, and the marked end of/
the shadow, *E*.

Figure 24

Now, in the first picture, the shadow 84:10
forms with the gnomon the acute angle/ *ABE*.
However, in the second picture it forms with it 11
an obtuse (angle)./ And so, if then the plane 12
is adjusted until *ET*, the distance of the head of
the shadow from the/ plane of the horizon, [is 13
known][1] then it is possible to determine from it
the desired shadow, I mean *BG*. That is/ that *AH* 14
will be known, and that it, in the first picture,
is the difference between/ the gnomon and the 15
distance of the shadow's head from the plane of
the horizon, I mean *B*[*H*][2], and in the second as
their sum. And [the sum of *TG*, the equation,
and][3] *EH*, which is equal/ to *TB*, the mean shadow, 18
as in the first picture, or the difference bet-
ween it and/ the mean shadow in the second picture, 19
is the desired shadow, I mean *BG*./ Hence, when 85:1
we have determined// the elevation of the f.210a
head of the shadow from the foot of the
gnomon, we subtract it (*HB*) from it (*AB*)/

---

[1]Text حتى هط ; read حتى علم هط as in the MS. In
the figure, *H* is restored from the MS.
[2]Text ج ; read ح . Here two lines are re-
peated in text and MS.
[3]Missing in the text; restored to make sense.

to obtain the (quantity to be) retained for divi- 85:2
sion. And so, if we take the (square) root of the
difference between the squares,/ of the actual shad- 3
ow and the elevation of its head, or its depres-
sion, it will be/ the mean shadow for that altitude 4
or depression. Then we divide the result by the
retained (quantity) [and multiply by the elevation
or depression of the shadow's head][1]; there/ comes 5
out the equation. If the retained (quantity) re-
sulted from the difference, we add the equation/
to the mean shadow, but if it was from the sum we 6
subtract the equation/ from the mean shadow. There 7
results, after addition or subtraction, the/ adjust- 8
ed shadow in the plane of the horizon, which is
what was desired.

Also, the ratio of *AH*, the retained, to *EH*, 9
the mean shadow, is/ as the ratio of *AB*, the gnomon, 10
to the desired shadow, it being *BG*. And so, if/
we multiply the mean shadow by the gnomon and we 11
divide the result by the retained, there results
the / adjusted shadow. If we measure *AE*, the hy- 12
potenuse of this shadow, with a thread or a ruler,/
and the square of the retained is subtracted from 13
the square of this hypotenuse, there remains the/
square of *EH*, the mean shadow. The ratio of *AH*, 14
the retained,/ to *HE*, the mean shadow, is as the 86:1
ratio of *AB*, the gnomon, to *BG*,/ the desired. 2
Hence if we multiply the mean shadow by the gno-
mon and divide the result/ by the retained, there 3
comes out the adjusted shadow. And when the alti-
tude of the sun at the time of/ measuring the shad- 4
ow of the gnomon *AB* has been determined, it is
possible for us to extract the inclination of the
adjusted shadow./ That inclination is equal to 5
the angle *EB*[*G*][2], and that is because the ratio of
EB to/ *BG* is as the ratio of the sine of angle *EGB*, 6
which is equal to the altitude, to the/ sine of 7
angle *EBG*. And so if we measured the shadow *EB*
and computed from its altitude the/ shadow *BG* we 8

---

[1]Missing in the text; restored to make sense.
[2]Text ر ; read ج .

compare the two, and if they are equal *EB* will be in the plane of/ the horizon. But if the computed is larger than the actual (shadow) the end of the shadow at *E* will be higher than/ (that) plane. But if the computed (shadow) falls short of the actual, *E* would be lower than/ the plane of the horizon. 86:8 9 10 11

For the determination of the amount of that elevation or depression, multiply the sine of the altitude/ of the sun for that time by its computed shadow, and divide the result by the actual shadow./ There will come out *ET*, the sine of the angle of inclination, to an amount such that with it *EB* is the/ total sine, and the ratio of *ET* to *EB*, in units of the sine, is as the ratio of *ET*/ to *EB*, in units of the shadow. And so, if we multiply what results for us of the sine of the angle of/ inclination by the shadow, and we divide the result by the total sine, there will result the amount of/ the elevation of the head of the actual shadow, or its depression below the horizon plane, in the parts in which/ the gnomon is graduated. 12 13 14 15 16 17 18

Similar to this is what Abū Bakr Muḥammad b. ʿUmar b. al-Farrukhān sought in his/ zīj, about the determination of the shadow of a gnomon erected on the top of a physical sphere of known/ diameter if that shadow was cast on its surface at a time when the altitude of/ the sun is known, and this is the recital of it: 87:1 2 3 4

Determine the diameter of the sphere in digits of the gnomon, then add the gnomon to (half) the/ diameter of the sphere and multiply the result by the sine of the altitude and divide what results by/ the total sine. There results the retained (quantity). Then multiply also the sum of (half) the diameter of the sphere/ and the gnomon by the cosine of the altitude, and divide the result by the total sine. What/ results multiply by itself and subtract the result from the square of half of the diameter of the sphere,/ and subtract the (square) root of what remains from the retained. Multiply the remainder by the cosine of/ the altitude, and divide the result// by half the diameter of the sphere, and make what results/ an arc directly (i.e. find its arc sine) to obtain (? a hiatus in the MS) which is the shadow on the back of the sphere/ in units of its great circle, the three hundred and sixty (one). If you want it in digits, multiply the diameter of/ the sphere by three and a seventh, and what results (will be) in parts of the arc of the shadow. Divide the result/ of that by three hundred and sixty; there come out the digits of the shadow on the back of the sphere. 5 6 7 8 9 10 11 f.210b 87:11 12 13 14 15

In this computation, because of the copyists, I have become confused concerning/ its proof. So, let us leave what we are not sure of and treat of what we know. Let/ gnomon *AB* (in Figure 23) be perpendicular to the surface of the sphere. Now, in the first picture (it is) on/ the convexity, but in the second picture it is on the concavity. The center of the sphere is *E*,/ and we draw *AG* tangent to it at *A*, and let the ray of the sun passing/ through the head of the gnomon be *KBH*, and 16 17 18 19 88:1 2

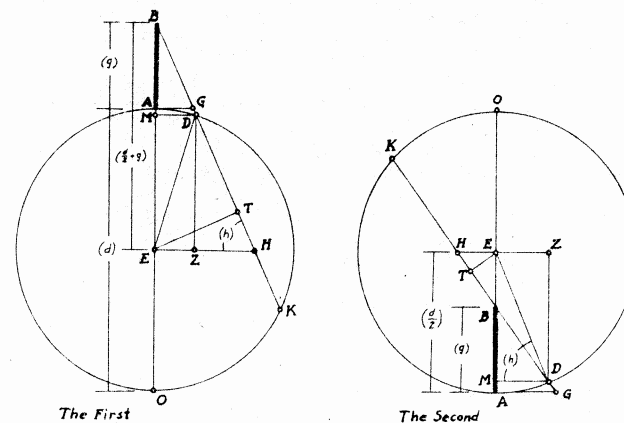



Figure 23

let the shadow of the gnomon on the plane of the 88:2
horizon be/ *AG*, and on the curved surface *A*[*D*][1]. 3
we erect *ZEH* [perpendicular to][2] the/ half-diameter 4
AE, and we drop perpendicular *DM* to *AM*. So it will
be the/ sine of the arc of the shadow, and perpendic- 5
ular *DZ* to *EH*, (will be) its cosine. The ratio of/
EB, the sum of the digits of the sphere's half-dia- 6
meter and the digits of the gnomon, to *BH*/ is as 7
the ratio of the sine of the altitude, which is
measured by angle *A*[*G*]*B*[3], to/ the total sine, I mean 8
angle *BEH*, the right angle. And so *BH* is known,/
and the ratio of its sine to *EH* is as the ratio of 9
the total sine to the cosine of/ the altitude, which 10
is measured by *EBH*. So *EH* is known and triangle/
BEH thus is known as to sides. We drop, in right 11
triangle *BEH*, the/ perpendicular *ET* upon its hypot- 12
enuse. And so the square of *EH* will be equal/ to
the product of *BH* by *HT*. And *HT* hence will be 13
known, and the perpendicular to it,/ *ET*, will be 14
known, and the half-diameter *ED* will be the hypot-
enuse of a right triangle with it and *TD* as legs./
So *TD* also will be known. And all of *HD* will be 15
known, and the ratio of *HD*/ to *DZ* will be as the 16
ratio of *HB* to [*B*]*E*[4]. And so *DZ* will be known, and
it is the cosine of/ the parts of the shadow on the 17
sphere.

The essence of its computation is that we add 18
the digits of half the sphere's diameter to/ the 19
gnomon. It will be the first retained quantity,
and we multiply it by the total sine. We divide
what/ results by the sine of the altitude, and 89:1
there results the second retained quantity. We
multiply it by/ the cosine of the altitude and di- 2
vide what results by the total sine; there results
the third retained (quantity)./ We multiply it 3
by itself and divide the result by the second re-
tained; what results is the/ fourth retained. We 4
square it and the third retained and take/ the 5

---

[1] Text ه ; read د as in the MS.
[2] Text قريبا، ; read قائما، as in the MS.
[3] Text ح ; read ج .
[4] Text نه ; read به.

difference between the two results, subtracting 89:5
it from the square of half the sphere's diameter/.
We add the root of what remains to the fourth re- 6
tained, and multiply the sum by the first/ retained, 7
and we divide it by the second retained, and we
transform what results by/ multiplying it by the 8
total sine. We divide what results by half the
sphere's diameter times/ [three][1] and a seventh, 9
and we divide what results by three hundred and
sixty, and there comes out the digits of/ the shad- 10
ow on the back of the sphere, or on its inside.

Also, since the product of the sum of *ET* 11
with half the diameter times/ the difference be- 12
tween the two equals the product of *DT* times *TK*,
hence *KD*/ will be known. The product of *KB* times 13
B[*D*][2] is known because it is equal to the/ prod-
uct of *OB* times *BA*, since the product of *KB* 14
times *BD*/ plus the square of *T*[*D*][2] equals the 15
square of *TB*, and the product of *KD* times/ *DB*
plus the square of *DB* equals the product of *KB* 16
times [*B*]*D*[3]./ And so, if we add to the product 17
of *KB* and *BD* the square of *TD*, there results/ 18
the square of *TB*. The difference between its
side (i.e. square root) and *TD* is *DB*,/ whose ra- 19
tio to *DM* is as the ratio of *BH* to *HE*. And so
DM, the sine of/ arc *AD*, is known. Its computa- 90:1
tion after obtaining the difference of what is
between the two previous amounts is that/ we
take its (square) root and increase it by half 2
the sphere's diameter in (one) place, and we
subtract the one from the other/ in a second 3
place. We multiply the one in one place by the
one in the other, then by four. We retain/ the 4
(square) root// of what results, and we multi- f.211a
ply the sum of the diameter of the sphere and

---

[1] Text ثلاثمائة ; read ثلاثة.
[2] Text ج ; read د.
[3] Text نر ; read بر.

the gnomon by the diameter of the sphere,/ and (also half) the retained root by itself. We take the (square) root of their sum, and subtract from it/ (half) the retained root, and multiply the remainder by the third retained, and divide the result by/ the second retained. There comes out the sine of the parts of the arc of the shadow, and we transform it into/ digits as in what preceded. 90:5 6 7 8

THE SIXTEENTH (CHAPTER) 90:9

ON THE DETERMINATION OF THE NOON SHADOW FOR ANY ASSUMED DAY 10

If the day is fixed, then the position of the sun/ at noon will be known. In order to proceed from the declination to the required, use as a means the determination of/ the noon altitude. A southern declination is not affected by its magnitude, since/ between it and the complement of the latitude of the locality, which is equal to the altitude at noon of the first (points) of/ Aries and Libra, is always the altitude of the sun on the noon of that day in the/ southerly direction. 11 12 91:1 2 3 4

However, as for a northern declination, it depends on the latitude of the locality, and is therefore divided into three/ kinds. One of them is when it is less than the latitude of the locality, whereupon the sum of it and the complement of the/ local latitude will be the noon altitude in the southerly direction. 5 6 7

The second (occurs) when it exceeds the local latitude, whereupon the sum of it and the complement of/ the local latitude, subtracted from a hundred and eighty, will be the noon altitude of the sun in a/ northerly direction. The third (occurs) when it equals the local latitude, whereupon the noon altitude will be/ ninety degrees, associated neither with north or south. The/ noon altitude at the time of zero declination will be the complement of the local latitude itself, and a/ separate chapter has been written about it. 8 9 10 11 12 13

However, as for the first kind, of the northern declination kinds, it prevails in the inhabited part (of the earth)./ But the second kind is peculiar to regions known as those having two shadows, because the head of/ the shadow will be opposite in direction to the altitude and if it is 14 15 16

possible in a single locality/ that the noon al- 91:17
titude should be at one time south and another
time/ north, the head of the noon shadow (will be) 18
one time north and the other time/ south. 19

As for the third, it is located in regions 92:1
having two shadows in between/ these two times 2
(above-)mentioned. It will also be the beginning
of regions having a/ single shadow in localities 3
whose latitude equals the inclination of the eclip-
tic, since its shadow will disappear once/ in a
year, at the summer solstice. Then the head of 4
the shadow during/ the rest of the time will be 5
toward the north. The terrestrial equator is among
the localities having two shadows,/ and the noon 6
altitude at it will always be the complement of
the solar declination. When the altitude is/ known 7
the ascendant also will be known, from what has
preceded.

This is the true law, supported by proof. 8
He who transgresses it,/ verily he forsakes preci- 9
sion in favor of simplification and approximation,
like the Indians, and they/ extract it by extremely 10
weak operations.

What I heard about them is that they find 11
a number associated with/ each latitude, differing 12
for different (localities), which they call [*as-*
tarki?][1] and it, for the region of Sind,/ whose lat- 13
itude is less than thirty parts, is thirty-six;
and for the region of Lahore,/ whose latitude is 14
about thirty-two, is thirty-eight, as though it is
the minutes (of daylight) of/ the longest day,// f.211b
or perhaps it exceeds it by one minute. They
subtract from it/ the minutes of the assumed day, 16
and multiply the second (i.e. the latter) by the
minutes of the night of that day,/ and we divide 17
the result by the minutes of that day, and there
comes out the digits of the noon shadow.

Abū Saʿīd al-Sizjī explained that he noticed 18
that some of the Indians multiply/ six by six and 19
(thus) obtain the base for Sind, and it is the
longest day. Then they multiply/ the difference 93:1

---

[1] Text استوكى ; MS استركى .

between the longest day and the assumed day, in 93:1
minutes, by five,/ and divide the result by four, 2
and he claims that the result will be the noon shad-
ow and this is/ what most of them do about it. 3

However, a minority of them follow in their 4
zījes the true (method) as we/ explained, but it is 5
difficult to demonstrate that a method is false be-
cause/ what is invalid cannot be correct except by 6
chance, and such/ coincidences are found here and 7
there.

Among such (methods) is what is said about 8
it that it is by doubling the solar declination.
If the declination is/ southerly its double is di- 9
vided by fifteen, and what results is added to fifty-
seven,/ and so the result of that will be, after 10
addition or subtraction, the noon shadow./ Simi- 11
lar to this is what is in the zīj of Abū ʿĀṣim
ʿIṣām the freedman of Kh[ā]lid[1] b. Barmak about it,/
and it is that he said,/ 12

"Take for each part of a northern declination 13
thirteen and two-thirds minutes, and subtract that
from the/ shadow of Aries for your locality. What 14
remains is the shadow at noon/ on such and such a 15
day. (But) [take][2] for each part of a southern dec-
lination twenty-five/ minutes, and add that to the 16
shadow of Aries for your locality and there results
the noon shadow".

But what is even more [crude][3] than this is 17
their saying, "Subtract the time-degrees of the day-
arc/ from two hundred and sixteen always, and divide 18
what remains by five and a quarter, and retain what
comes out./ Then divide the difference between the 19
day-arc and a hundred and eighty by eighteen,/ and 94:1
what comes out, add it to the retained quantity, and
there results the noon shadow." These things/ re-
sult from experience valid for one (particular) po- 2
sition but not another, nevertheless both of its
parts were taken as universally (valid).

---

[1] Text خلد ; read خالد.
[2] Text جدد; read خذ as in the MS.
[3] Text طلالا ; read ضلالا.

However, as to the statement of some of 94:3
them, "Divide by the sine of the noon altitude/
nine hundred and seventy-five, and multiply what 4
results by itself, and subtract from it/ forty- 5
two and a quarter. Take the (square) root of
the remainder, and it will be feet of the/ noon 6
shadow", it does not belong to this type, but
rather it is the enunciation of a valid rule.

That is because it has been made evident 7
that the ratio of the sine of the altitude to the
total sine is as the/ ratio of the gnomon to the 8
cosecant, and the product of the total sine by the
gnomon does not/ change its value. So if the to- 9
tal sine is a hundred and fifty, and the gnomon
is six/ and a half, the number resulting from the 10
multiplication of one of the two by the other is
the number which is supposed/ to be divided by the 11
sine of the altitude. The result of it will be
the hypotenuse of the shadow (cosecant), the legs
(of the right triangle) being/ the gnomon and the 12
desired shadow.

THE SEVENTEENTH (CHAPTER) 49:13

ON THE EQUINOCTIAL SHADOW FOR ANY LOCALITY 14

The equinoctial shadow is the noon shadow 15
when the sun is/ in the first (point) of the sign 16
of Aries, or the first of the sign of Libra, and
thus it is one of the noon shadows, with the/ con- 17
dition that (the sun) have no declination. When
this is the case it will be the cotangent of the
complement of/the local latitude, the equatorial 18
shadow.

That is why al-Nayrīzī and Yaʿqūb b. Ṭāriq 19
said, concerning its determination, "Multiply the
sine/ of the latitude of the locality by the gno- 95:1
mon, and divide the result by the cosine of the lo-
cal latitude, and there results/ the equatorial 2
shadow".

There is some doubt as to the words of 3
Yaʿqūb, because he calls the sine a straight chord,/
just as in the words of al-Battānī who calls sines, 4
chords so that he halves them/ by bisection. With 5
regard to this shadow, its magnitude can be obtained
by observation, so that/ it replaces the local lat- 6
itude. Indeed, the Indians delimit localities by
it, as we delimit them/ by means of their latitudes. 7

Concerning it al-Kindī has a detailed state- 8
ment in which he says "The shadow of the head of
Aries/ is shorter than the shadow of the head of 9
Libra.// And the shadows from two opposite f.212a
places/ in the signs are unequal except at five 10
degrees of each of the signs,/ Virgo and Pisces". 11
The meaning of his saying is similar to what we
pointed out due to the differences/ in the distances 12
of the sun from the earth.

Let *ABGD* (in Figure 26)[1] be the meridian 13
circle with center *E*/ which is the (center of the) 14
universe, and *EZ* is the common part between its

---

[1] In the text figure, on p.99 and out of order, ح is missing.

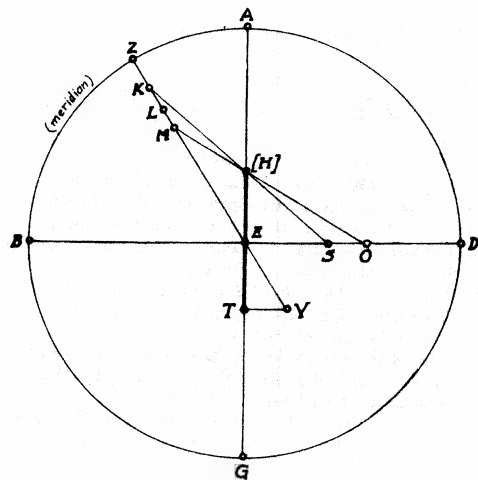



Figure 26

plane and the plane of the celestial equator./ 95:14
And we mark off *EL* (as) the mean distance of the 15
sun from the earth, and because the apogee,/ in 16
the opinion of Ptolemy, according to him (al-Kindī),
is at five and a half degrees of Gemini, hence/
the first (point) of Aries is between the mean dis- 17
tance and the apogee, and its distance from the
earth (there)/ is greater than the mean distance. 18
Let it be *EK*. In like manner the first of/ Libra 96:1
will be between the mean distance and the (point)
opposite to the apogee, and its distance from the
earth (there) is/ less than the mean distance. 2
Let it be *EM*. However, at the position where/ the
gnomon is *ET*, the shadow at each one of the two 3
points/ *K* (and) *M* will be *TY*, constant in amount, 4
and it is necessarily existent/ and perceptible. 5

But al-Kindī in his reference to the 96:6
variation of the shadow at the two (points) takes/
the gnomon as *EH*, and he passes from the two points 7
K (and) *M* at its head the two rays/ *KHS* (and) *MHO*. 8
Thus the shadow varies at the two/ (above-)mentioned 9
points by the amount *SO*. But that is the result
of fancy regarding the orbit of/ the sun. For *EL*, 10
according to Ptolemy, is a thousand and a hundred
(read two hundred) and ten times the/ radius of the 11
earth. Can anyone tell me how many times the gno-
mon this will be? But/ the two mean distance (po- 12
sitions) are not at all far distant from the equi-
noxes, especially by/ our time. The amount *MK* com- 13
pared to *EL* is insignificant, and will not be no-
ticed in the/ solar orbit, because of the minute 14
amount of the earth's radius and the double (of
the distance)/ between the center of the pareclip- 15
tic and the eccentric compared to its radius. But/
this situation is perceptible for the lunar orbit 16
because half the/ diameter of the earth is not so 17
insignificant compared to its (the orbit's) radius,
and (because of) the magnitude of the difference
between its nearest/ and farthest distances. 18

However, as for what is said (both) to the 97:1
common people and the learned, that, "If the equa-
torial shadow increases by/ a digit in the direc- 2
tion of the Daughters of the Great Bear (*Ursa
Major*), then it has riṣen by a hundred and twenty
farsakhs,/ but if it increases by one digit in the 3
direction of the inferior *Suhayl* it is depressed
in it by the same amount",/ it has come, I think, 4
from some of the Manicheans, who have the idea that
north (implies)/ elevation and that south (implies) 5
depression and corruption.

The first is due to (the fact) that travel 6
on the earth is along arcs, and there is no relation/
between arcs and rectilinear (objects) such as 7
chords or shadows.

The corruption (i.e. depression) of the 8
second (occurs) when it (the shadow) increases to
the limit and (then) decreases/ by receding from 9
it, unless they consider the recession of the head

of the shadow a break for it in the southerly direction./ But the word increase (as used) by them is misleading in their interpretation of it. 97:9 10

For this southerly increase is impossible in the inhabited part (of the earth) unless we assume/ for it a position to the south of the terrestrial equator. So let *W[N]FC*[1] (in Figure 25) be on/ the meridian, and *WAY* the common part between its plane and the/ plane of the celestial equator. 11 98:1 2 3

Figure 25

[1]Text ز ; read as in the MS. In the text figure, for ن read و ; for ر read ن ; ى and س are missing.

The center of the universe is *A*, and we assume *WN*, *NF*, (and)/ *FC* equal, each one cutting off on the earth a hundred/ and twenty *farsakhs* and representing them. However, as for the terrestrial equator, on it the/ equatorial shadow disappears, and it will be (non-zero) at positions differing[1] from it in latitude. So we extend/ *NA* along its length until *AB*// becomes equal to the gnomon. And we erect to it/ perpendicular *BG*. So it will be the equatorial shadow at latitude *WN*;/ we extend *FA* along its length until *AE* also becomes equal/ to the gnomon, and we erect to it the perpendicular *EDT*, and it will become the equatorial shadow at/ latitude *W[F*. We extend *CA* along its length until *AZ* becomes equal to the gnomon, and we erect to it the perpendicular *ZHY*, and it will become the equatorial shadow at latitude *W]C*[2]. 98:4 5 6 7 f.212b 8 9 10 11

It is apparent that triangles *ABG*, *AED*, and *AZH* are equal, and so the [increments][3] of shadows *BG*, *BK*, and *BM*, for arcs *WN*, *WF*, and *WC*, which/ have equal increments, are not equal. I mean that *GK* is not equal to/ *KM* because if we extend perpendicular *GSL* with *GS* (and) *SL* equal/ because of the equality of the two angles *[G]AS*[4] (and) *SAL*, and if we extend *SO*/ parallel to *ML*, *MO* would be equal to *OG*, and so *KG* is/ smaller than *KM*, and so (the assertion) that the equatorial shadows differ by a digit in each hundred/ and twenty *farsakhs* is false, to the praise of God. 12 13 14 15 16 17 18 19

[1]Text النحنية ; MS المتحية.
[2]Missing in the text.
[3]Text فصول ; read فضول.
[4]Text ح ;read ج.

THE EIGHTEENTH (CHAPTER) 99:1

ON THE CORRECTION OF THE MERIDIAN DIRECTION 2

BY TWO (EQUAL) SHADOWS, OR BY TWO EQUAL AZIMUTHS

As to the [construction][1] of a surface on the 3
face of the earth parallel to the horizon, and level-
ling and/ adjusting it, it is a matter which concerns 4
the craft of plastering and whitewashing. The prac-
titioners of it have instruments/ with plumb lines 5
and weights which guide them in obtaining it. The
coming to rest of a smooth sphere on any/ part of it 6
(the surface), the uniform flow of water off it, and
the even rolling of mercury on it/ are the most trust- 7
worthy indications of its perfection and correctness.

Let *ABG* (in Figure 27), under such circumstan- 8
ces, be on the common part between the planes of the/
meridian and the horizon, so it will be the meridian 9
line, and *SEO*, being/ on the common part between the 10
planes of the celestial equator and the horizon,
(will be) on the east-west line (*lit.* the line of/
the equinox). Let *HDB* be the day triangle, in which 11
HD is the sine of the/ noon altitude and *DB* is the 100:1
(algebraic) sum of its cosine and [*E*]*B*[2], the sine
of the/ rising amplitude; and *HB* is the day sine. 2
Let *TKG'* be the/ time triangle, and we join *K* (to) *E*. 3

We extend it to *Y*, unlimited. The shadow will 4
be/ at *EY*, and its azimuth will be distant from the 5
east-west line by the amount of the angle *SEY*,/ and
from the meridian line by the amount of angle *BEY*. 6
And we make angle/ *OEC* equal to the angle *SEY*, and 7
we extend *CE*/ until it intersects *KM* at [*L*][3]. 8

---

[1] Text لصف ; read نصب as in the MS.
[2] Text م ; read ه . The appearence of the figure has been altered considerably, but by using the primes in our version, little violence is done to the text. For the lower ى read ص .
[3] Text لو ; read ل as in the MS.

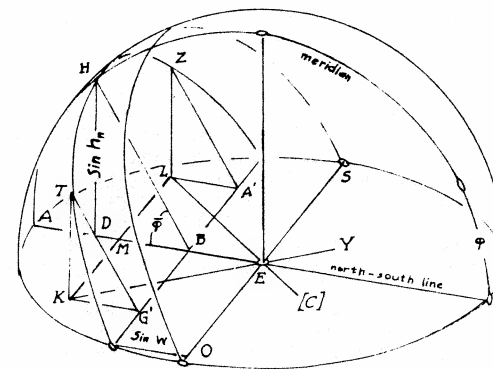



Figure 27

We draw a perpendicular (through) *L* to *AG*, 100:9
and we make angle [*LA'Z*][1] equal/ to angle *K*[*G'*]*T*[2], 10
and *A'Z* equal to [*G'*]*T*[2] and parallel to it. And we
join *Z* (to) *L*./ And since *ME* is common to the two 11
similar triangles *KME* (and) *LME*,/ the two similar 12
triangles *LSE* (and) *KOE* are equal. And because *A'L*/
(and) *A'Z* are equal to the corresponding (lines) *G'K* 13
(and) [*G'*]*T*[2], and the angles *A'* (and) *G'* are equal,/
hence the two bases *KT* (and) *LZ* are equal, and the 14
one triangle is equal to the (other) triangle/ and
similar to it, and the two parts of the shadow at 15
the two times *T* (and) *Z* are equal, and the altitude
at them/ has the same amount because *ZL* (and) *KT*, 16
their sines in the two of them are equal. But the
shadow is linked with/ the altitude. And so the 17
two shadows also are equal, and the distances of

---

[1] Text الز ; read لاز.
[2] Text ح ; read خ.

the two times from/ noon are equal because *KM* 100:18
(and) *ML*, their sines in the solar daily path,
are equal.

Hence the (time) past of daylight at time 101:1
T will be equal/ to that remaining of it at time 2
Z. And it is known that when we obtain// two f.213a
azimuths, in the two halves of/ the day, equidis- 3
tant from mid(day), then the meridian line will
necessarily be in the middle/ between them, just 4
as *EB* is midway between the two azimuths *EK* (and)
EL. There results the bisection of the/ angle *KEL*, 5
or that the east-west line makes equal angles with
the two,/ like *KEO* (and) *LES*. So there result by 6
our construction *EK* and *EL*,/ equal. We join *K* (and) 7
L and draw *OS* parallel to/ *KL*. And because the 8
azimuth, the altitude, the shadow, the time past
of the day(light), and the remainder/ of it are 9
dependent on each other, and if all of them are
in/ one direction from noon, of the two directions, 10
east and west, and in one locality, and at/ one 11
time, then they have fixed amounts, and/ will not 12
change under similar conditions. So is the case
(also) for differing directions from noon if/ the
daily circle is (a single) one, or if the two 13
daily circles are bounded by equal declination(s),
there is concurrence in direction (from noon).

Let, for example, *ABGD* (in Figure 28) be 102:1
the plane of the horizon of an assumed locality,/
and *AE*[*G*][1] on it is the meridian line, and *BED* 2
the east-west line,/ and *AS*[*G*][1] the circle of 3
the meridian, and the zenith on it is *S*.

We put/ on it and at a distance equal to the 4
complement of the altitude some almucantar *KOZ*.
The sun on it would be/ at point *O*. And we pass 5
through it from point *S* a great circle,/ *SOT*. So
OT will be the altitude of the sun, and *TB* (will 6
be) the distance of its azimuth/ from the equinox. 7
And we describe about *Y*, the pole of the celestial
equator and at a distance *YO*,/ the complement of 8
the solar declination, the small circle *OL*. And

[1]Text ب ; read ج.

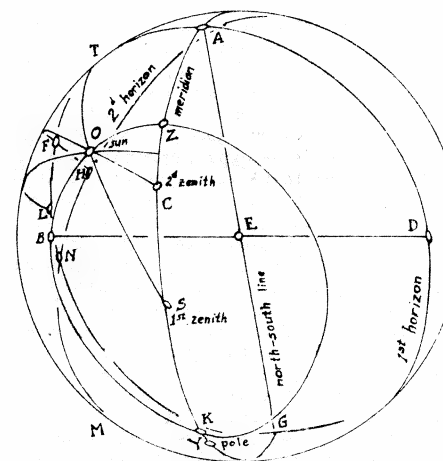



Figure 28

OL will be, on the small circle, (the measure of) 102:8
what has passed/ of the day, if the direction is east, 9
or what remains of the day if it is westerly./ And
LB is the rising amplitude of the sun or its setting 10
amplitude. And since all points on the almucantar/
KO[*Z*][1] determine altitudes equal to the altitude of 11
OT, hence all/ the daily circles which cut this al- 12
mucantar will have its/ altitude, but at points other 13
than point *O*.

But parallel daily circles do not intersect, 103:1
and the azimuth of *TB* will not be/ on this almucan- 2
tar except at point *O*, and thus (likewise) the circle
LO. As for/ the shadow, it is known that its amount 3
is determined by the amount of the altitude, and its

[1]Text ر ; read ز as in the MS.

(the shadow's) azimuth is the opposite of its 103:3
(the altitude's) azimuth./ And so if the shadow 4
of the altitude *OT* is found in many daily circles,
its azimuth for them will/ not be opposite to 5
point *T* on the horizon, not even when the (time)
passed is/ to the amount of *LO*. For, let *C* be the 6
zenith of some other locality,/ and *H*[*N*]*M*[1] is on 7
its horizon. We extend *COH*, and the altitude of/
O will be on it, and it is *OH*, greater than *OF*. 8
OF is/ greater than *T*[*O*][2] because angle *OTF* is a 9
right (angle), and (together) with/ the circle it 10
(*TO*) will be less than *OL*. And so the altitude *OT*
cannot obtain at the locality/ *C*, except (if) on 11
an almucantar equal to the almucantar *KOZ*. And
so the circle/ through it will be smaller, and here 12
[is][3] joined to the shadow, by its being added to/
the altitude. But one is not astonished at the 13
shadow as he is astonished at the altitude because
of (the fact that)/ the azimuth is determined once 14
the shadow is determined, as well as the east-west
direction together with/ the altitude only. 15

And when it is asked and said, is it possi- 16
ble, in a locality of known latitude, that/ the
ascendant may be one thing at two different times, 17
with two different positions of the sun,/ but its 18
altitude in one direction// and of one amount, f.213b
the usual way of answering/ is to hasten to deny 19
its necessity, and the statement about it con-
cerning shadows is that/ the ascendant and one 104:1
of the two shadows will be equal in one of the
quadrants of the horizon, but at two/ different 2
times.

In order to explain that, let *ABGD* (in Fig- 3
ure 29) be the local horizon quartered by the
lines of the (cardinal)/ directions. *A* is the 4
east and *B* the south, and *EZH* half of the/ eclip-
tic, with pole *T*. [*E*][4] will be the degree of the 5

---

[1] Text ب ;read ن.
[2] Text ح ; read ع.
[3] Text صحت ; read اصحبت.
[4] Blank in the MS. On the figure, for ط read ك as in the MS. ص، is missing in the text, present in the MS. For the • at the zenith in the text, read س. ي is missing in the text.

Figure 29

ascendant, and we draw *TS*,/ the circle of the 104:6
latitude of visible climate. And *S* on it is the
zenith, and we draw with it (as pole) the/ almu-
cantar *K*[*M*][1]. Its intersection with the ecliptic 7
will be at (the points) *K* (and) *M*/ in one direc- 8
tion, which is east in the example. Their alti-
tudes,/ *KN* (and) *ML* are easterly and equal. And 9
it is known that the sun, if it is at/ *M*, its 10
elevation will be *ML*, perpendicular to the hori-
zon, but if it is at/ *K*, its altitude will be *KN*, 11
and the two are equal, because they are on a single
almucantar./ The ascendant at both times is one 12
(and the same), and it is point *E*. And the sun
in between the two/ times will travel on the e- 13
cliptic along the arc *MZK*. And if the/ almucantar 14
were not the one intersecting the ecliptic on one
side, but on the contrary on two (different) sides,
like the almucantar/ *OY*, then the altitudes of the 15

---

[1] Text س ; read م .

sun at their two intersections, namely *O* (and) 104:15
Y would be equal./ They (the altitudes) would 16
be *OF* and *YC*, and the ascendant is the same, and
what is between the two positions of the sun is/
greater than (in) the first (case). 17

When the rising amplitude of the ascen- 105:1
dant is southerly the two altitudes *SL* (and)/
KZ will be on the western side, as they were there 2
on the eastern. But at their vanishing/ one of the 3
two will be eastern and the other western neces-
sarily, but we have no need of these special ca-
ses,/ except that in which (it) will be in one 4
place.

As for the two equal altitudes, they are 5
of no use to us except for/ the two equal azimuths 6
which accompany them, and the shadow is the indica-
tion for them.

As for the two equal arcs of revolution, 7
their utility is like the utility of the two equal
altitudes/ only. The equality of the distances 8
of the two times on the two sides of/ noon has no 9
indication except the equality of the two altitudes
or the equality of the motions in the two times./
And (for determining) the equality of the motions, 10
one resorts to the instruments with which time is
measured/ by the outflow of water, or sand, or 11
other substances having similar particles or by
the inflow of/ water into them. The equality 106:1
of the two altitudes (is determined) by obser-
vation with the armillary sphere and the astro-
labe (? *al-ṣafā'iḥ*),/ or it is deduced from the 2
shadow which is associated with it. Verily, if
we observe the altitude with/ the proper instru- 3
ments, both morning and evening, we can determine
the line/ midway between the two azimuths of the 4
shadow, and this will be the meridian line. If
we observe the two equal shadows,/ that would be 5
the operation known as that with the Indian circle,
and indeed it is related/ to them, because (it is) 6
in the Arkand Zīj, and the zījes of the Indians,
and their computations, and such things were the
first to enter/ the domain of Islam. Its cons- 7
truction is to set up the gnomon perpendicular to/

a plane surface made parallel to the horizon, 106:8
such as the gnomon *AB* (in Figure 30) and we de-
scribe about/ center *A* and at any distance desired 9
a circle, the wider and the greater the circumference/
the more accurate would the operation with it be. 10

Then we observe the shadow in the first 11
half// of the day, and it is extended in the f.214a
direction of/ the west, decreasing until it enters 12
the circle. And the place of its entrance is
marked/ on the circumference. Let it be, for 13
example, [*H*][1]. Then we observe in the other half
of the day,/ and it increases, extending itself in 14
the direction of the east until it goes outside the
circle at a point/ *D*, for example. And thus are 15
found two equal altitudes, and the meridian is/
necessarily between the two. We join *H* (to) *D* by 16
a straight line. Then either we bisect/ chord *HD* 17
at *E*, or arc *HD* at *T*, or we complete them/ to a
whole circle at *Z* and we join from center *A* to any 18

Figure 30

[1]Text ج ; read ح.

of the midpoints/ *Z*, *E*, or *T*, or (we connect) 106:19
all of them by the line *ZT*. And so the meridian
line will bisect/ angle *HAD*. But Pulisa the 107:1
Greek and Vijayanandin of Benares describe about/
both *H* (and) *D* and at a distance *HD* a circle, and 2
we join the head of the/ resulting fish-shaped 3
figure from the intersection of the two circles, to
its tail. And that line will be along *ZT*.

Then, if we want, we extend for the east- 4
west line either a perpendicular from *A*/ to *ZT* or 5
a diameter from the midpoint of one of the two
halves, *ZDT* (or) *Z*[*H*]*T*[1]./ And if we desire, after 6
obtaining *ZT* and *HZ* we erase the remaining (lines)
and we describe about/ center *H*, at any distance we 7
desire, a circle, and its diameter which is along/
ZT, will be the meridian line, while that which is 8
along line *HD*, will be the east-west line./ How- 9
ever, the approved amount for the gnomon for this
operation is that whose shadow in/ the winter will 10
be shorter than the half-diameter of the circle in
all inhabited (regions), lest its length be incon-
venient/ and fail to reach the circle, and it 11
passes from the western region to the eastern
region/ outside of it. 12

Ptolemy limited the inhabited parts on the 13
north by an island, [Thule][2],/ claiming that its 14
latitude is sixty-three parts. And the complement
of its latitude will be/ twenty-seven parts. The 15
altitude of the first point of Capricorn there is
three parts and a quarter and a sixth of a/ part. 16
The digits of its shadow will be two hundred and
one and a quarter digits. That is/ sixteen and 17
three quarters times the gnomon. And when the
half-diameter of the/ circle is made more than 18
seventeen times the gnomon, the end of the shadow
will be (quite) distant,/ and the matter becomes 19
[difficult][3].

However, we say that the nations in whom 108:1
we find/ enough humanity to notice the virtue of 2

[1] Text ج ; read ح.
[2] تولي supplied from the MS; missing in the text.
[3] Text عشر ; read عسر as in the MS.

considering (religious) codes and who rejoice 108:2
zealously in science are/ those whose abodes do 3
not exceed forty-eight parts of latitude, and the
complement of/ this latitude is forty-two parts, 4
and the altitude of the first point of Capricorn
at it is/ eighteen parts and a quarter and a sixth 5
of a part. The digits of its shadow are thirty-
six digits/ and three tenths of a digit[1], and that 6
is close to three times the gnomon. And so it is
evident/ that if the gnomon is made equal to an 7
eighth of the diameter the shadow would not fail
in this latitude/ at the winter solstice, from 8
penetration into the circle. Had it not been for
the nation known/ as the Bulgars, who are Muslim 9
and are located quite far to the north, a lati-
tude of/ forty-five would have sufficed, and a 10
gnomon equal to a sixth of a diameter (likewise).

Abū Bakr Muḥammad b. ʿUmar b. al-Farrukhān 11
tries to use, in his zīj,/ a sixth of a diameter
once, and half a sixth another time. And he 109:1
who understands his rule, which/ we presented (be- 2
fore, finds that he) takes for every locality an
amount for the gnomon which he continues to use,/
but he might use, for what he mentions concerning 3
the Indian circle, another (amount, *or* method?).

It is that we [erect][2] perpendicular *AB* (in 4
Figure 31) to a plane parallel to the horizon, and
at its/ head a ruler [*GZ*][3] is made to turn about 5
in all the regions// parallel to the horizon,/ f.214b
and at its end *G* (is) a sight *GE*, (and) attached 6
to its base a plumb line [*G*]*M*[4]/ whose [pointed][5] 7
end touches the face of the earth. Then the rul-
er is rotated in the morning until/ the eye of 8
the sun is opposite the sight which shades the
middle of the ruler. It is as though the shadow
at that time/ was [*G*]Z. And we mark the position 9

[1] Text اصابع ; read اصبع.
[2] Text نصف ; read نصب.
[3] Text حد ; read جر as in the MS.
[4] Text ح ; read ج.
[5] Text محدد ; read محدب.

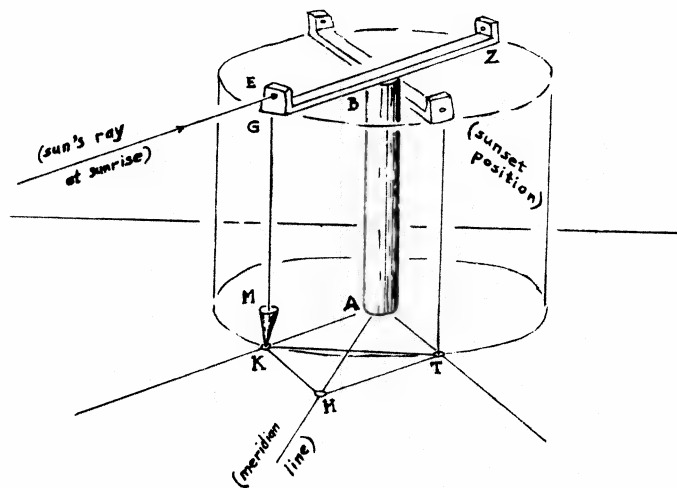



Figure 31

of the end of the plumb line *M* on the ground/, 109:9
and it is *K*. Then, in the evening we rotate it 10
until the sun's eye is also opposite the target,/
and we observe the shadow of the target until it 11
reaches *Z*. We mark at that time the position of
the end of the/ plumb line on the ground; let it 12
be *T*. And by determining the two points *K* (and)
T/ verily we have singled out two of the azimuth 13
lines through the center, the two equal lines *AT*
(and) *AK*/ corresponding to two times at which the 14
solar altitude will be equal. We join *K* (to) *T*,/
and we construct on *KT* an equilateral triangle *KTH*, 15
and we join/ *A* (to) *H*, thus dividing angle *KAT* 16
into two halves. But noon is between/ these
two times, and its line is the middle one between 17

the two azimuths. And so *AH*/ is the meridian 109:18
line. If it happens that points *K*, *A*, (and) *T*
are on/ one straight line, the operation would be 19
for the time when the altitude has no azimuth (i.e.
when the sun rises due east).

We may make a balance sidewise on its hor- 110:1
izontal ruler (*ʿamūd*) which will control/ the shad- 2
ow of the tongue. Then we weight its two pans
with two equal weights so that the ruler is adjust-
ed/ parallel to the horizon, and we observe at some 3
time during the first half of the/ day the shadow 4
of the tongue so that its end reaches along the
middle of the vertical (? *ʿamūd*). We mark the end
of the/ shadow, and at the two points of tangency of 5
the bottoms of the pans with the plane of the ground,
and that is why/ the two pans are made in the shape 6
of a cone so that the eye can determine the two
points/ (above-)mentioned. And if we join them 7
there will result/ the position of line *AT* (in Fi- 8
gure 32). The operation is like this (also) after
noon, and we observe/ so that the end of the shadow 9
of the tongue comes at the middle of the vertical,
like the first amount/ marked on it.

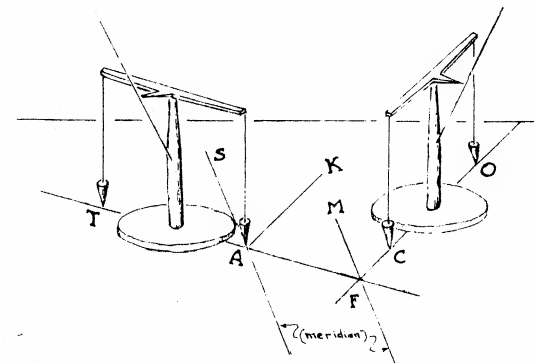



Figure 32

We mark also at that time the two places 110:11
of tangency of the pans on the ground, and we join/
the two points of tangency,and there results the 12
position of line *KA* or one parallel to it. If/
the two latter points of tangency are *O* and *C*, 111:1
then *OC* will be parallel to *KA*./ We extend both 2
TA and *OC* until they intersect at [*F*][1],/ and [*F*]*M*[2] 3
will be the angle bisector of *OFT*, the meridian
line./ Also *AS* is the meridian line for the one 4
locality. In the event that they are not/ far
apart they will be parallel, although in fact they 5
are/ great circles on the face of the earth inter- 6
secting at the axis of the prime (i.e. celestial)
sphere. Since they are perceived as/ straight lines, 7
they are parallel on it, and each of them is, at a/
single locality, the meridian line for the gnomon 8
set up at it, and that is what we wanted.

When it happens that one (sic, read *neither*) 9
of the two lines *OC* (and) *KA* intersect/ the exten- 10
sion of line *A*[*T*][3], then that altitude at which
the measurement was taken is the altitude/ which 11
has no azimuth. And everything we explained for
simplifying the operation is summed up as the/
objectives of the (operation with the) aforemen- 12
tioned Indian circle.// f.215a

---

[1] Text ب ; read ف . In the figure there are two ص's; for the upper one read س . For ف (?) read ب .

[2] Text ع ; read ف .

[3] Text د ; read ط .

THE NINETEENTH (CHAPTER) 112:1

ON THE CORRECTION OF THE MERIDIAN LINE 2

It is evident from what has preceded with re- 3
gard to the Indian circle, that operations with it are
confined to a single/ almucantar. But it is not 4
restricted to a certain almucantar and no other
except by reason of the/ greatness or smallness 5
(i.e. size) of the circle (which has been) drawn.
And so, if the greatest of the almucantars,/ namely 6
the horizon, is sought, the shadow at the time of
sunrise extends infinitely. Thus we do not/ need 7
(anything) but its intersection with the circumfer-
ence of the circle. And when the sun sets/ the shad- 8
ow extends infinitely also. The meridian line will
be (midway) between the intersection of/ the two 9
shadows with the circle because of the equality of
the two distances from noon in (point of) percep-
tion, without/ extreme precision, because the sun 10
does not rotate with the motion of everything in a
small circle parallel to the celestial/ equator,
but by virtue of eastward motion it describes a 11
spiral line/ because of which its rising amplitude 12
for that day differs from its setting amplitude.
And so we do not take its daily path (***madār***) as
being/ parallel (to the equator) except for approxi-
mation, and for restricting (ourselves) to percep- 13
tion.

The greatest difference between two (suc- 14
cessive) points of intersection of this spiral and/
the almucantar will be at the horizon and around 15
the times of the two equinoxes because of the mag-
nitude of/ the difference in the declination, to 16
the extent that on the day which is after the vernal
(equinox) or before/ the autumnal it will be a bit 17
more than a fifth of a degree. And on the day which
is before the vernal (equinox)/ or after the autum- 18
nal, a bit less than a fifth of a degree. It is

known that the duration (of the sun)/ above the 112:19
earth increases the amount of this difference if it
(the rate of change of the difference) were to re-
main constant./ But it can decrease in amount 113:1
simultaneously with increase due to the duration
(of daylight), and equivalence of the two/ effects[1] 2
at some northerly distance is known (to occur).
When this (distance) is exceeded the decrease van-
ishes and is overcome./ The difference continually 3
decreases as the summer solstice is approached,
until it becomes seconds/ without any minutes. 4

However, in the southern half, the [dif- 5
ference][2] of the declination and the tarrying above
the earth are/ both decreasing from (the time) of 6
the autumnal equinox until the winter solstice, but
they are increasing/ together in the quarter which 7
follows it. The situation with the almucantars is
the same, because every/ one of them has a share in 8
this difference, but the distance of their inter-
sections from/ noon, when it is least, this dif- 9
ference will be less and more subtle./ The meridian 10
line obtained by it will be nearer to its true pos-
ition, and that is because it deviates from (cor-
rectness)/ in accordance with the deviation of these 11
two intersections from parallelism with the celes-
tial equator. And so its/ southern end falls be- 12
tween the south and the east so long as the sun is
in the declining half/ from Cancer to the end of 13
Sagittarius, and it falls between the south and the
west if it/ is in the rising half, from the first 14
(point) of Capricorn to the end of Gemini./ It can 15
hardly be exactly at the very south except on a day
on which the solstice occurs at/ noon. Inexactness 16
in it evades perception on days in/ the vicinity of 17
the two solstices, and that is because of the small-
ness of the difference mentioned (above), which
appears on the other (days) if one/ increases the 18
circle used in the operation.

I was making observations in Khwārazm in or- 19
der to determine the declination by (use of) shadows

[1]Text الحالين ; MS الحالان .
[2]Text فيفاضل ; read فتفاضل .

with a circle/ whose diameter was fifteen cubits. 114:1
The results were leading to impossibilities, and I
was perplexed by it/ until I came upon the reason, 2
namely the deviation of the meridian line and the
east-west line from their (true)/ positions. So I 3
corrected it and it became valid.

Pulisa, in his *siddhānta* refers to this 4
meaning, and the determination of the (time) passed/
of the day at each of two times: the entry of the 5
shadow into the circle, and its going out from it./
He determines the true solar longitude at the two 6
(times), extracts its declination, and prescribes
the multiplication of the difference between the
two declinations/ by the minutes of days between the 7
two times, the division of the result by sixty, the
multiplication of/ what results from the division 8
by the radius in digits of the circle described on
the ground,/ and the division of (that) result by 9
the total sine. He claims that what comes out is
the digits between the actual/ point of exit of the 10
shadow from the circle and the point opposite// f.215b
the point of entry,/ and it is the true point of 11
exit, which will be to the south if the sun is in
the descending/ half, and to the north if it is in 12
the ascending half.

But I think that in this operation there is 13
a corruption on the part of the translator, for it
requires/ that the ratio of this quotient to the 14
difference between the two declinations shall be
equal to the ratio of the minutes of days/ between 15
the two times to sixty, for he knew that the devia-
tion of the line/ joining the point of entry of the 16
shadow to its point of exit from parallelism with
the east-west line is/ proportional to the difference
in declination at the two times. According to this 17
law it is necessary that/ this difference between 18
these two times shall be to the change in declina-
tion for that entire/ day as the minutes between 19
the two times are to sixty. This requires the ex-
traction of the position of/ the sun at the time 115:1
of entry and its position after it by one complete
day, and the declinations of these two positions./

Then we multiply the difference between the former two by the difference between the two times, and we divide the result by [sixty][1] and there comes out/ the desired difference in declination. But its determination from the two declinations at the solar positions/ for the times of entry and exit is easier and better, and that is Pulisa's operation where he satisfied himself/ with the difference between the two declinations, neither multiplying nor dividing by anything. Better/ than this would be to extract the azimuth of the sun at the two times, then to use the difference between them/ instead of the difference in declination. 115:2 3 4 5 6 7

I think that Pulisa was aware of the relation between azimuth and/ rising amplitude. His country is low in latitude, where the amounts of the declination and/ the rising amplitude are close (to each other), and so he approximated by taking the difference between the two declinations instead of (that) between the two azimuths. 8 9 10

[1]Blank in the MS.

THE TWENTIETH (CHAPTER) 115:11

ON THE EXTRACTION OF THE MERIDIAN LINE BY THE USE OF THREE SUCCESSIVE SHADOWS 12

By virtue of what has been translated for us of the sayings of Brahmagupta, the son of Jisnu, that if three shadows/ are measured on one of the two sides, east or west,/ of one gnomon, and their ends are marked, and if three circles are described about them intersecting each other, there result/ two fish(-shaped figures), one of them from the intersection of the first with the second and the other from the intersection of/ the second with the third. If the head of each one of them is joined to its tail and/ the two lines are extended in the direction of their intersection, and if (further) their point of intersection is joined to the/ foot of the gnomon, then this connecting line will be the meridian line. 13 14 15 16 17 18 19

An example of it is that *E* (in Figure 33) is the foot of the gnomon and *AE* the longest of the/ three shadows, and *EG* the shortest of them, and *EB* the middle one of them, and the two fish(-shaped figures) resulting/ from the circles are *DAHB* (and) *ZBTG*. Their common chords are *DH* (and) *ZT*,/ and the intersection of the two chords is *K*. So *KE* is along the meridian line, and what results from/ these two fish(-shapes) is the extension of the two perpendiculars from the midpoints of the two lines/ *AB* and *BG* which are *M* (and) *S*. However, of the chords of the circle, it is evident/ that these two intersect at the center of its diameter, (which is) supposed to be on the axis (*sahm*). But as for the chords of the hyperbola,/ it (the point of intersection of the perpendicular bisectors) will not fall on the axis except by chance. 116:1 2 3 4 5 6 7 8

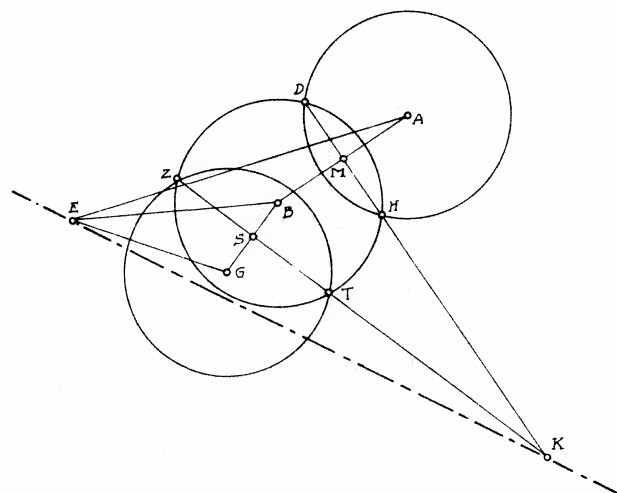



Figure 33

When I suspected the translator, I obtained a strong impression of his ineptness for this operation, together with the confusion of/ ideas. An urgent need might arise for obtaining the meridian line during one of the two halves of the day (light) of a/ certain day, without waiting to observe the shadow in the other half; I changed over/ to an operation based on what is in the book (the) "Analemma" of Diodorus. 116:9, 10, 11, 12

It is that we describe about center *E* and at a distance equal to the length// of the gnomon/ a circle *YMS* (in Figure 34), (and we erect) a perpendicular *EZ* to *AE*. We cut off arc *ZH*/ equal to arc *YM*, and arc *TH* equal to arc *MS*./ 117:1, f.216a, 2, 3

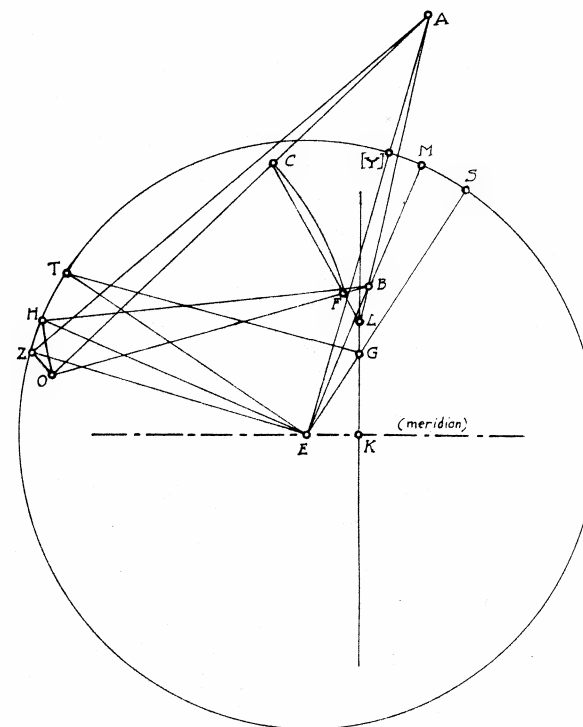



Figure 34

And we connect *A* (and) *Z*, *B* (and) [*H*][1], [*G*][2] (and) *T*, and we describe about center *A* and at a distance/ *AZ*, and about center *B* and at a distance 117:4, 5

[1]Text ع ; read ح as in the MS. In the figure, ر is missing in the text, restored from the MS. ى is missing from the text.

[2]Text ع ; read ج

BH, two circles intersecting at/ *O*. We connect *O* (with) *A*, (and) *O* (with) *B*, and we describe about center *O* and at a distance/ *GT*, arc *FC*. And we extend the two lines *CF* (and) *AB* until they/ intersect at *L*. We join [*G*][1] (to) *L*, and we drop on it the perpendicular *KE*. It will be/ along the meridian line. 117:6 7 8 9

Let the proof of the correctness (of this) be (that) the gnomon is *ZE* (in Figure 35). The triangles/ *AEZ*, *B*[*E*]*Z*[2], and [*G*]*EZ*[3] are triangles of the shadows at the times of/ the three observations. And *AZ*, *BZ*, and *Z*[*G*][1] are their hypotenuses, and (they are) on the surface of the shadow cone/ whose vertex is the head of the gnomon. It is evident that the parts in common between the plane of/ any circle perpendicular to the axis of the shadow cone and between the plane of the (conic) section which/ is formed by the head of the shadow on the horizon (plane) will be parallel to the plane of the celestial equator, because/ the circle (is) parallel to it, and the axis of the (conic) section is the meridian line. And because *Z* is/ the vertex of the cone and the circle [*G*]*FC*[1], the parallel (one) is at a distance of *ZG*, it is one of/ those circles which is parallel to the celestial equator. We drop two perpendiculars *CS* (and) *FM*/ to the plane of the horizon and falling on the two lines *AE* (and) *BE*, and because *AZ* is greater/ than *ZB*, and *CZ* (and) *FZ* are equal, so *AC* is greater/ than *BF*. And the ratio of *AC* to *CZ*, I mean *AS* to/ *SE*, is greater than the ratio of *BF* to *FZ*, I mean *BM* to/ *ME*. We extend *ST* parallel to *AB*, so the ratio of *AS* to/ *SE* is as the ratio of *BT* to *TE*. The ratio of *BT* to *TE*/ is greater than the ratio of *BM* to *ME*, and so *BT* is greater than *BM*./ And angle *ASM* is part of angle *AST*. We make angle *SAB*/ common (to both), and the two angles *ASM* and *SAB* are less than the two angles *AST* (and)/ *SAB*. But the two angles *AST* (and) *SAB* are equal to two right 10 11 12 13 118:1 2 3 4 5 6 7 8 9 10 11 12 13 14

---

[1] Text ح ;read ج.
[2] Text ص; read ه.
[3] Text حزص; read حهر as in the MS.

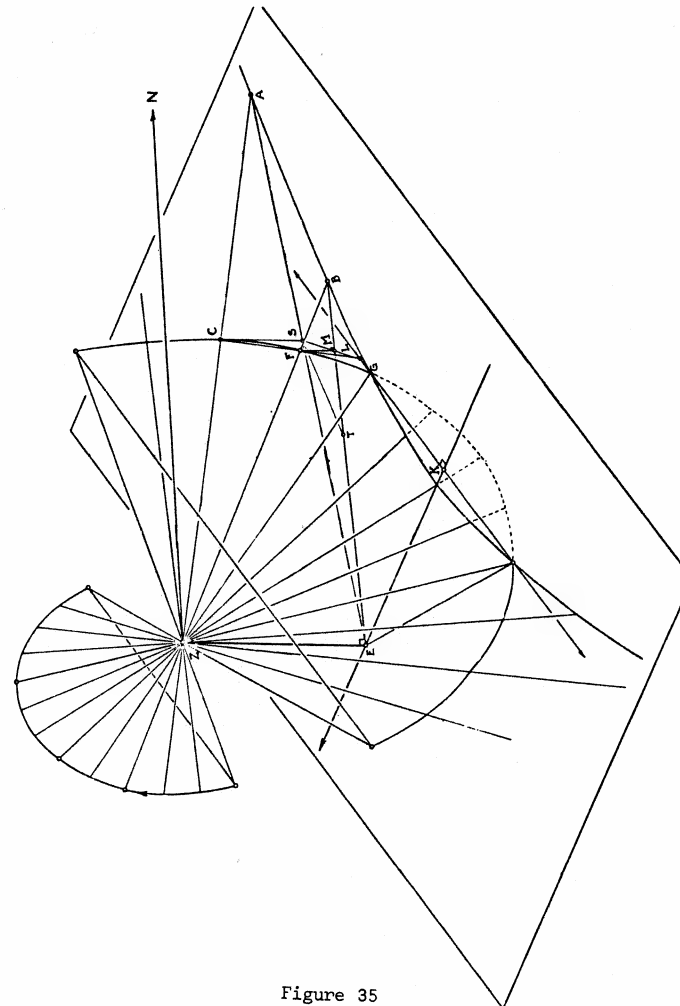



Figure 35

angles,/ and the two angles *ASM* (and) *SAB* are 118:16
less than two right angles, and so the two lines
SM (and) *AB*/ intersect in the direction of *MB*. 17
Let them intersect at *L*. And because the two lines
SL (and)/ *AB* are in the two planes *CSMF* (and) *ACFB*, 18
hence *L* is along/ the common part. But *CF* also is 19
in these two planes between the two,/ and so 119:1
points *C*, *F*, (and) *L* are on a single straight line.
But *L* is in/ the plane of the horizon and in the 2
plane of the circle [*G*]*FC*[1], and so it is on the/
common part between the two. And point *G* thus is 3
in the plane of the horizon and this circle./ So
line [*G*]*L*[1] is along the common part between the two, 4
and it is parallel to the celestial equator, and
the/ meridian line is perpendicular to it. But 5
point *E* is on this line, and so the perpendicular
EK is/ the meridian// line. f.216b

Then we turn to the first figure, the cons- 7
truction (figure), and we say that *A*[*Z*][2], *BH* (and)/
[*G*]*T*[1] are hypotenuses of the shadow triangles, I 8
mean (those) corresponding to *AZ*, *BZ*, and *Z*[*G*][1],/
and that *AO* there is equal to *AZ* here and *BO* there 9
is equal/ to *BZ* here. And their two bases *AB* are 10
equal. So triangle *ABO*/ there is the triangle *ABZ* 11
here, and the two lines (*OC* and) *OB*/ there equal the 12
two lines *ZC* (and) *ZB* here./ And so *OC* in both of 13
the two is one (line), and likewise *AC* and [*B*]*F*[3].
And the acute (triangles)/ *CFB* in the two are simi- 14
lar, and their corresponding [sides][4] are equal,
and *CF*/ intersects *AB* at *L*. 15

And so the distance of the intersection from
B for both of them is the same./ The point *L* in 16
the two figures is the same, and the placing of [*G*]*L*[1]
is similar. The/ assumption of one side (of the 17
day) is not essential, but we should see to it that
(we have) a longest/ and a medium (shadow) for 18
what we did. For when two of them are equal the me-
ridian line will bisect them,/ and it will bisect 19
the angle which is bounded by them, whereupon the
affair is reduced to (that of) the Indian circle.

[1] Text ح ; read ج .
[2] Text د ; read ر as in the MS.
[3] Text ن ; read ب .
[4] Text اصلاها ; read اضلاعها as in the MS.

THE TWENTY-FIRST (CHAPTER) 120:1

ON THE EXTRACTION OF THE MERIDIAN LINE 2

BY ANY ONE SINGLE MEASUREMENT WHATSOEVER

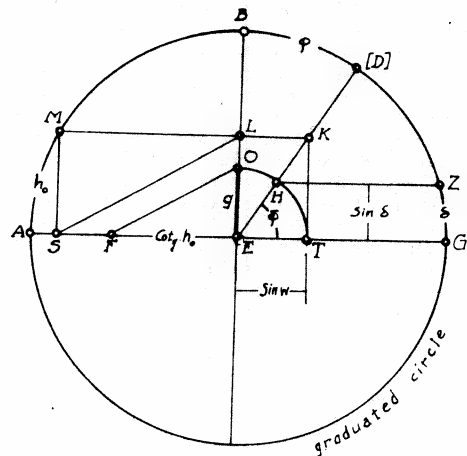
Such a topic as this has many [aspects][1], and 3
among them (i.e. the methods) is that we divide the
circle laid out/ into three hundred and sixty equal 4
parts; and each part is (sub-)divided to the amount
possible in/ minutes, and a gnomon is set up on its 5
plane, and we observe the direction of the sun's
rising at the/ rising of half its body from under 6
the earth, or the direction of its setting at the
sinking of/ half its body. That is that we observe 7
the passage of the middle of the gnomon's shadow
across the circumference of the circle,/ and we put 8
a mark on it. Then we compute the rising amplitude
of the sun if we had obtained/ the mark for its ris- 9
ing, or its setting amplitude if we had obtained it
for its setting, and we determine/ its direction. 10
Then we measure off from this mark its equal in the
opposite direction. And it is evident that/ the 11
position with which we finished is one of the two
ends of the east-west diameter, and the diameter
perpendicular to it/ is the meridian line. Another 12
(method) is that we suppose a number for the azi-
muth whose occurrence is possible/ on that day. 13
Then we extract for it the amount of the shadow, and
we describe about the base of the gnomon/ and at 14
the distance of that shadow a circle, [and][2] we
observe the shadow of the gnomon at its entry or
exit from/ this circle. When its end reaches its 121:1
circumference we extend the diameter passing
through the middle of/ the shadow to the circum- 2
ference of the graduated circle, and we measure off
from the end (an amount) equal to that/ azimuth (in 3

[1] Text وجود ; read وجوه as in the MS.
[2] Text او ; read و as in the MS. Lines 15 through 19 in the printed text are repeated in lines 1-5 on the following page 121; they are suppressed in the translation.

a direction) opposite to (its) direction, and it also will fall at one of the two ends of the east-west diameter. 121:3

Another (method) is that the graduated circle be *ABG* (in Figure 36), with / center *E* and *EB* perpendicular to the diameter *AG*, and we assume on it/ [*B*]*D*[1] equal to the latitude of our locality and [*G*]*Z*[2] equal to the declination of the sun/ at the time (in question). We join *D* (to) *E*, and we extend *ZH* parallel to [*G*]*E*[2],/ and we draw *ET* equal to *EH*//. We extend *TK* parallel to/ *EB*, and *KLM* parallel to [*G*]*A*[2]; and *MS* parallel to/ *BE*. And we join *L* (to) *S* and cut off *EO* to the amount of the gnomon set up/ at *E*. We extend *OF* parallel to *LS*, and we describe about *E*/ and at a distance [*E*][3]*F* a circle. We observe, in one of the two halves of 4 5 6 7 8 f. 217a 10 11 12

Figure 36

[1]Text ز ; read ب .
[2]Text ح ; read ج .
[3]Text ع ; read ه .

the day, the entry of the shadow,/ and [its exit][1] from this small circle, and we pass from it a diameter in it which will be the/ east-west line, and the diameter perpendicular to it will be the meridian line. 121:13 14

Its proof is: let [*G*]*T*[2] (in Figure 37) be on the common part between the two/ planes, of the horizon and the small circle, and [*E*][3] the center of the horizon (circle) and *EZ* along the/ east-west line. Let the day-triangle be *ABG*, and the time-triangle/ [*H*]*ZT*[4] provided that *Z* is on the east-west line, then/ *ZH* is necessarily the sine of the altitude having no azimuth. We erect perpendicular *ED*/ to *BG*, and it will be the sine of the solar declination, and *EG* is the sine of its/ rising amplitude. The two angles *G* (and) *T* are equal to the complement of the latitude of the locality. And the angles 122:1 2 3 4 5 6 7

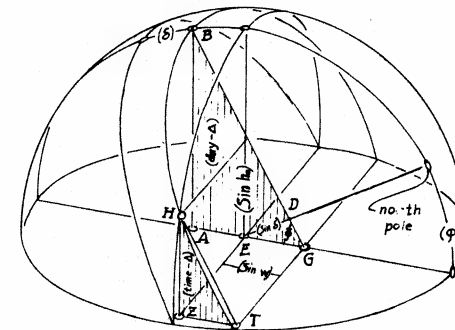



Figure 37

[1]Text نجرجه ; read مخرجه as in the MS.
[2]Text ح ; read ج .
[3]Text ح ; read ه as in the MS.
[4]Text ج ; read ح .

B[*GE*][1]/ (and) [*H*] *T* [*Z*][2] are equal to the lati- 122:8
tude of the locality. And so the ratio of *ED* to
EG will be as the ratio of the/ cosine of the lo- 9
cal latitude to the total sine, I mean the ratio
of the sine of angle *G*/ to the sine of the right 10
angle *D*. And the ratio of *ZT*, the sine of the ris-
ing amplitude,/ to *ZH* the sine of the altitude, is 11
as the ratio of the sine of the local latitude to
its/ cosine, I mean the sine of angle *H*, to the 12
sine of angle *T*.

When we make the construction figure [*GZ*][3] 13
will be the declination of the sun, and/ *GD* the 14
complement of the latitude of the locality, (and)
ET, I mean *EH*, is the sine of the rising amplitude./
And *TK*, the sine of the altitude having no azimuth, 123:1
the ratio of *ET* to it/ is as the ratio of the sine 2
of *BD*, the latitude of the locality, to the sine of
[*G*]*D*[4], its complement. And/ *LE* equals *KT* and so *SE* 3
is the cosine of this altitude, and the ratio of the/
sine of the whole altitude to its cosine is as the 4
ratio of the gnomon to its shadow at that time,/ and 5
the ratio of *LE* to *ES* is as the ratio of *O*[*E*][5] to *EF*.
And verily/ we supposed that *O*[*E*][5] equals the gnomon, 6
and *EF* is the shadow of this altitude.

It is known that if (this actually) takes 7
place (at the time of) the observation it would be
along the/ east-west line, since it has zero azi- 8
muth. But this is [familiar?][6] as being/ a part- 9
time (phenomenon) because it does not occur for
southern declinations, but only for northern decli-
nations./ Similarly, what comes before it (the 10
operation) is restricted to a certain time of the
day, the occurrence of which/ is to be watched 11
carefully. But perhaps the meridian line is required
immediately, there being no/ time for waiting. 12

---

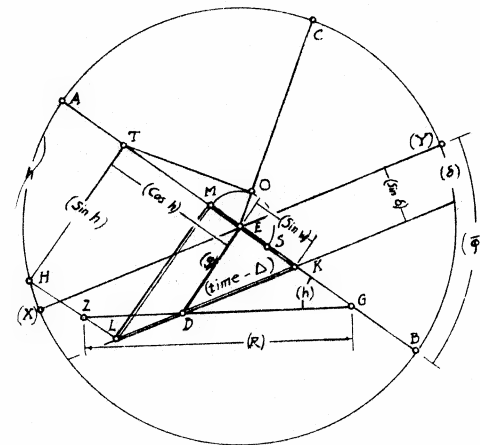
[1]Text ح ; read جه.
[2]Text حط ; read حطز.
[3]Text جد ; read جر.
[4]Text ح ; read ج.
[5]Text س ; read ه.
[6]Text ماوف ; read مالوف.

Therefore we work out an operation for any time 123:13
whatsoever, without/ waiting for another.

Let the position of the shadow at it be *EG* 14
(in Figure 38) and we extend it in two directions/
along its own length so that there results from it, 15
in the graduated circle, the diameter *AB*. We erect
to it/ perpendicular *ED* equal to the gnomon, and we 16
join *DG*, and we make *GZ*/ equal to the radius *AE*, and 17
we pass through point *Z* line [*H*]*ZL*[1]/ parallel to *AB*, 18
and we join *E* (to) *K* (which will be) equal to the
noon shadow, and we extend/ *KDL*. We drop two per- 19
pendiculars *HT* and [*L*]*M*[2] to *AB*, and we describe
about/ center *E* and at a distance *EM* the semicir- 124:1
cle *MOS* in a/ direction opposite to the time, I mean 2
that if it is before noon, then (it will be) in the
direction of the motion of/ the sun, which is the west. 3

Figure 38

---

[1]Text ج ; read ح.
[2]Text ك ; read ل.

But if it is after it (i.e. noon), then (it will 124:3
be) in a direction against the motion of the sun,/
which is east. We extend// *TO* tangent to the f.217b
circle *MOS* at *O*,/ and we extend through its (point 5
of) tangency *EOC*. It will be along the/ meridian 6
line.

Its proof is that the ratio of *DE*, the gno- 7
mon, to *EG*, its shadow at the time,/ is as the ratio 8
of the sine of the altitude to its cosine, I mean
the sine of the angle *G* to/ the sine of the angle 9
EDG. And so *AH* is the altitude at the time if the
semi-/circle *AHB* is imagined (as being) perpendicu- 10
lar to the plane of the horizon, and *TE* is the co-
sine of this/ altitude, and at its position. But if 11
the semicircle *AHB* is/ the meridian circle, and it 12
is supposed that *EK* is the noon shadow, *KDL* would
be on/ the common part between the plane of the 125:1
small circle and the meridian plane, and the angle
K/ equals the complement of the latitude of the 2
locality. So triangle *KLM* is the time triangle in
amount, (but)/ not in position, because *AEB* is not 3
on the line of the meridian, and (because) the side
parallel/ to it in this triangle is not *KM*. It is 4
known that the cosine of the altitude is the hypot-
enuse of the triangle having as legs/ the two lines 5
in the horizon plane, from the foot of the vertical
dropped from the/ altitude, perpendicular respec- 6
tively to the meridian and the east-west lines. The
one of (these) two extending/ to the east-west line 7
is called the argument of the azimuth, and *EM* is
equal to it. So *OT* is/ the other one extending to 8
the meridian line, because *ET* as it is, is the hy-
potenuse of the right triangle of which the two
are legs./ But *TMO* is perpendicular to *EOC* because 9
it passes/ through the center to the (point of) tan- 10
gency. So line *EOC* is the meridian line/ which we 11
sought.[1]

---

[1]Text طلبناه ; MS طلبنا.

THE TWENTY-SECOND (CHAPTER) 125:12

ON THE AMOUNTS OF THE DAY AND NIGHT, AND 13
THE [DIFFERENCES][1] OF THE ASCENSIONS

It is evident to one who is acquainted with 14
the shape of the universe that variation in longi-
tude between/ east and west has no effect except in 15
the difference of rising or setting proportionate
to that/ variation; and that the other differences, 16
in rising and setting amplitudes, and the differ-
ences in/ noon altitudes and shadows, and the 17
difference between daylight and night, and suchlike
things,/ are of those which are caused by variation 18
in latitude between north and south.

Each one of the peoples follows, for the 19
determination of positions, (a method) other than
that/ followed by the others. Among them are 126:1
those who determine it by the altitude of the north
pole, which is equal to the (terrestrial) latitude,
and/ others determine it by the hours of the longest 2
day in them (i.e. in those places), as was done for
the division of the climates.

Among them are those who use for it the 3
farsakh, and other units by which/ distances are 4
measured.

Also among them are those who determine at 5
it (the place) the shadow of Aries. It is the noon
shadow on the day/ of equality (of day and night) 6
[associated with][2] the complement of the latitude.
Verily the daylight, throughout the whole year for
one place,/ differs from the night because of the 7
difference in rising times, in a manner related to
the difference of the noon shadow at it. Along
these lines/ the Indians operate in their use of the 8
shadow for determining times.

---

[1]Text فصول ; read فضول .
[2]Text التاسع ; read التابع as in the MS.

Thus spoke Brahmagupta in the Brahmasid- 126:9
dhānta, "To determine the equation of daylight/
multiply both the sine of the declination of the 10
star and the total sine by themselves (i.e., square
each),/ and take the (square) root of the differ- 11
ence between them. It will be half the diameter
of the daily path of the star./ Then multiply the 12
sine of the star's declination by the equatorial
shadow, and divide what results by/ twelve and 13
multiply what results by the total sine, and di-
vide the result by/ the radius of the star's daily 14
path. What results is the sine; find its corre-
sponding arc and it is, (in) *prāṇas*,/ the equation 15
of daylight". Convert it into [*vināḍī*]s[1] by di-
viding by six. Each sixty [*vināḍī*]s[1]/ is a [*gha-* 16
ṭī][2]. [Vijayanandin][3] did the same, but he brought
together the two multiplications, as well as/ the 17
two divisions. And so he said, "Multiply the ra-
dius of the sun's daily path by the gnomon, and di-
vide by/ the result the product of the equatorial 18
shadow and the sine of its declination, then (mul-
tiply) by the total sine,/ and there comes out the 19
(sine of the) equation in minutes of the day".

The proof of this (is as follows): Let 127:1
ABG (in Figure 39) be the meridian,/ and ***AG*** the 2
part in common between its plane and the plane of
the horizon, and ***BEH***// / the part in common f.220a[4]
between it and the plane of the celestial equator,
whose pole is ***T***. And let/ the common part be- 4
tween the planes of the meridian and of the star's
daily path be/ ***KM***, and extend ***TE***. ***SM*** will be the 5
sine of the equation of daylight in/ the daily 6
path, and we make ***EZ***, the perpendicular to ***AG***,
equal to the gnomon, and we extend/ ***ZH*** parallel to 7
AG. It will be the equatorial shadow. We drop per-
pendicular/ [*G*]*L*[5] upon ***TE***, and because ***AB*** is the 8

[1] Text بناری ; read بنادی as in the MS.
[2] Text كهزى ; read كهري as in the MS.
[3] Text حاتد ; MS حانند ; read بجانند .
[4] ff. 218 and 219 are missing,presumably due to an error in numbering,since the text is not interrupted.
[5] Text ح ; read ج .

complement of [*G*]*T*[1], so the two triangles/ ***BED*** 127:9
and ***ELG*** will be similar and equal, and triangle ***MSE***
will be similar to both of them./ So the ratio 10
of ***MS*** to ***SE*** will equal the ratio of ***HZ*** to ***ZE***,
but ***SE***/ is the sine of the declination, and ***EK*** is 11
the total sine, it being the hypotenuse of a right
triangle of which it (i.e., ***SE***) and ***SK*** are legs./
SK is the cosine of the declination, it being what 12
is called the radius/ of the star's daily path, 13
and its ratio to ***SM*** equals the ratio of the total
sine, I mean ***BE***,/ to what corresponds to it along 14
the radius of the sphere. This is the transfor-
mation, I mean transforming (the units of)/ ***SM***
from a unit in which ***SK*** is the total sine. Thus 15
is the sine of/ the equation of daylight made known, 16
and this is exactly what Yaʿqūb b. Ṭāriq explained/
in the book "On Causes" (*Kitāb al-ʿilal*). 17

For he said, "Take the reversed chord of the 18
distances of the end of each (zodiacal) sign (from
the equator) and subtract it from/ 3438, and there 19

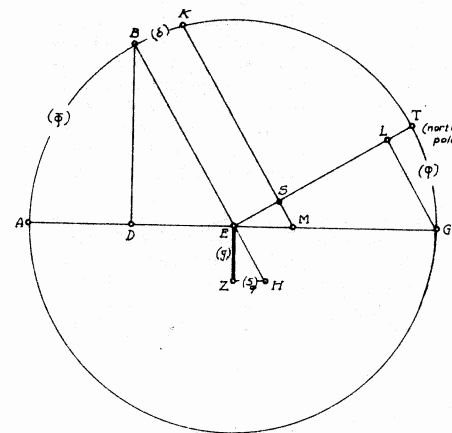



Figure 39

[1] Text ح ; read ج .

will remain the chord of the hoop of the sign's 127:19
daily circle, and multiply the straight chord of
this distance/ by the equatorial shadow, and di- 128:1
vide the result by the digits of the gnomon. Mul-
tiply what results by/ 3438, and divide the result- 2
ting amount by the chord of the hoop of the sign's
daily circle. That which comes out,/ make it an 3
arc, and it is the excess for Aries, and the de-
ficiency for Virgo". This is because the/ straight 4
chord to him is the ordinary sine, and the reversed
(chord) is the versed (sine), and this/ (above-) 5
mentioned number is the minutes of the total sine
according to [Āryabhaṭa][1], and the chord of the hoop/
of the sign's daily path is the cosine of its dec- 6
lination, I mean half the diameter of its daily
path, and the distance of the/ sign is its declina- 7
tion, and the (above-)mentioned excess and defi-
ciency refer to the differences between a/ right 8
ascension and an (oblique) ascension for the lo-
cality.

Since the arc sine comes out for him in 9
minutes,/ he claims that the result is in *prāṇas*, 10
that is "respirations", because with the Indians,
adjusted ones (respirations) are equal to/ a revolu- 11
tion of minutes of equatorial time-degrees, and
each six time-degrees make a [*ghaṭī*][2],/ I mean 12
one of the minutes of the day, whose seconds are
called [*vināḍī*][3], but/ the common people among 129:1
them call them [*jashaha* (for Sanskrit *caṣaka*), and
also *jakaha*][4].

Al-Khwārizmī set up a table in his zīj call- 2
ing it the "Differences of the Ascensions/ for 3
the Earth", and in it opposite each degree (is a
number) which if multiplied by the equatorial shadow
gives/ the sine of its equation of daylight, and 4
that is the product of the sine of the declination
of the degree divided/ by the cosine of its declina- 5
tion, multiplied by a hundred and fifty seconds.

---

[1] Text ارجيصد ; read ارجبهد as in the MS.
[2] Text لدءرى ; read كهرى as in the MS.
[3] Text بناری ; read بنادی as in the MS.
[4] Text حبشه والضاحكه ; MS جشه وايضا جكه .

For an explanation regarding its truth, 129:6
let *AG* (in Figure 40) be on the local horizon and *B*/
the ascension of a degree on it, and *ZGD* be of the 7
meridian, and *AEZ*/ a quadrant of the celestial e- 8
quator at *G*, and we extend from it two great circles
DA and *BE* (*DTA* and *DBE*)./ So *AE* will be the equa- 9
tion of daylight, and *BE* will be the declination of
the degree,/ and *BD* will be the complement of its 10
declination, and we extend great circle [*Z*]*BT*[1], and
BT will be/ what is called in our books a mean, and 11
the ratio of the sine of [*G*]*Z*[2] to the sine of [*G*]*D*[2]/
equals the ratio of the sine of *EB* to the sine of 12
BT. But the ratio of the sine of/ *ZG*, the altitude, 13
to the sine of *GD*, its complement, equals the ratio
of the gnomon to/ the shadow. And hence the ratio 14
of the sine of *EB* to the sine of *BT*, equals the ra-
tio/ of the gnomon to the equatorial shadow, and 15

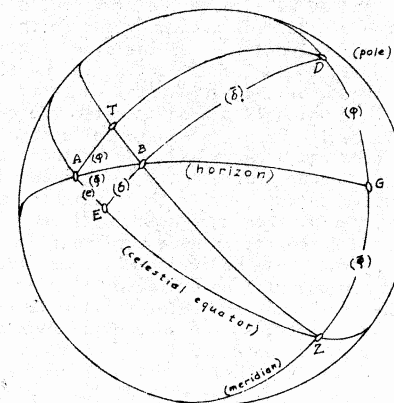



Figure 40

---

[1] Text د ; read ز as in the MS.
[2] Text ح ; read خ.

the ratio of the sine of ***BT***// to the sine of ***BD***/ f.220b
equals the ratio of the sine of ***AE*** to the sine 129:16
of ***DE***, the quadrant. And verily the ratio/ of the 17
equatorial shadow to the sine of ***ED***, the total sine,
is a compound (ratio), and if we equate the parts/
of the total sine and the parts of the gnomon, that 18
equates the third and sixth of these six/ quantities, 19
and it will be that the ratio of the shadow of Aries
to the sine of ***BT*** equals the ratio of/ the gnomon 130:1
to the sine of ***BE***, and the ratio of the sine of ***BT***
to the sine of ***AE***/ equals the ratio of the sine of 2
DB to the sine of ***ED***, that being equal to the gno-
mon. And so by the equality/ of the disturbed ratio, 3
the ratio of the shadow of Aries to the sine of ***AE***
will be equal to the ratio of/ the sine of ***DB*** to the 4
sine of ***BE***. If we multiply the equatorial shadow
by/ the sine of the declination of the degree, and 5
we divide what results by the cosine of its dec-
lination there comes out (the sine of)/ the equation 6
of daylight. And if we advance the division, di-
viding the sine of the declination by the cosine of
the declination,/ there comes out the sine of the 7
equation of daylight when (the quotient is) multi-
plied by the shadow of the terrestrial equator.
Hence it/ is that which is placed opposite the de- 8
gree in the table, in case the equatorial shadow is
measured/ in (the same) parts as parts of the total 9
sine, but it is measured by a scale (*or* gnomon) di-
vided into twelve parts. So if/ the total sine is 10
taken as sixty parts, then the gnomon will be a
fifth of it, and hence/ it is necessary that the 11
thing placed in the table be five times that divi-
dend/ so that harmony will be achieved. 12

But the total sine according to al-Khwārizmī 13
is two and a half parts, so the gnomon is/ four and 14
four fifths times it, and hence it is necessary
that what comes out from the/ division of the sine 15
of the declination by its cosine be divided by four
and four-fifths/ in order that harmony be estab- 16
lished. Division by four and four-fifths is the
taking of a part of/ twenty-four parts of it, and 17
everything which is required to be divided by twen-
ty-four can be multiplied by a hundred and fifty

seconds, I mean a part in twenty-four of sixty 130:17
minutes,/ giving the desired (thing), and that is 18
what we sought.

In the book "On Causes" (***Kitāb al-ʿilal***) 131:1
of Yaʿqūb ibn Ṭāriq (it says) multiply the equatorial
shadow/ by the chord of the increment for Aries, 2
namely the sine of its right ascension, and divide
what results/ by the noon shadow in the position of 3
the maximum ascension of the signs at the terres-
trial equator,/ it being twenty-six digits and fif- 4
ty-eight minutes, and we mean by that the equatorial
shadow/ for the position whose latitude equals the 5
inclination of the ecliptic. So there comes out the
chord of/ the deficiency of the zone of Aries, and 6
the increment of the zone of Virgo; make it an arc,
and it will be the/ decrease for Aries and the in- 7
crease for Virgo.

The basis of this operation is that the ratio 8
of the sine of the equation of daylight of the degree
(of solar longitude) to/ the sine of its right ascen- 9
sion equals the ratio of the equatorial shadow for
the locality to/ the (shadow of) the complement of 10
the inclination of the ecliptic, I mean the equato-
rial shadow for the latitude which is equal to the
inclination/ of the ecliptic. And so let us put 11
down on the picture what we need for it, letting
OBH (in Figure 41) be a quadrant/ of the ecliptic, 12
and ***HK*** the inclination of the ecliptic, and ***HL*** be
along the parallel of Cancer./ We extend ***DLM***, and 13
so ***LM*** will be the inclination of the ecliptic, and
AM the equation/ of its daylight, and it is known 132:1
what we will do concerning it later on. The ratio
of the sine of ***AE*** to/ the sine of *A*[*Z*][1] equals the 2
ratio of the tangent of ***BE*** to the tangent of ***ZG***,
and the ratio of the/ sine of ***AZ*** to the sine of ***AM*** 3
equals the ratio of (the tangent) of ***ZG*** to the tan-
gent of/ ***LM***. By the equality, the ratio of the 4
sine of ***AE***, the equation of daylight of the degree,
to the sine of/ ***AM***, the solsticial equation of day- 5
light, equals the ratio of the tangent of *E*[*B*][2],

---

[1]Text د ; read ز .
[2]Text ز ; read ب .

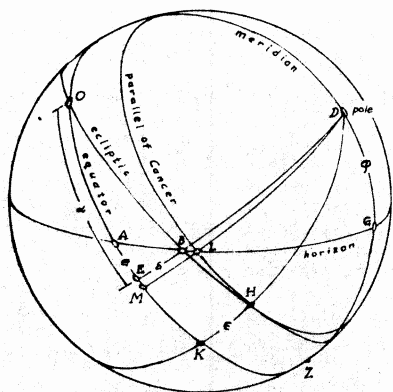



Figure 41

the declination of the degree, to the tangent 132:5
of/ *LM*, the inclination of the ecliptic. But the 6
ratio of the tangent of *E*[*B*][1]// to the tangent f.221a
of *HK*,/ which is equal to *LM*, is the ratio of the 7
sine of *OE*, the right ascension,/ to the sine of 8
O[*K*][2], the quadrant, and that is what we wanted to
explain.

There are found in the Indian zījes operations 9
for the extraction of the differences of risings/
for the heads of the signs by means of the shadows 10
extracted from readings/ by approximation, since 11
there is no (linear) relation between the shadows
and their arcs themselves, but without their sines,/
and we will relate it after introducing these quan- 12
tities computed accurately for the assumed latitude,/
to be compared with their results in order to 13
differentiate (between) the nearest of them to the
truth and the most distant of them.

[1]Text ز ; read ب .
[2]Text م ; read ك .

Let that latitude be twenty-four parts, and 133:1
the digits of the equatorial shadow/ at it 5,21 and 2
the equation of the daylight of Aries, I mean, the
end of it, 5;[13][1],/ and the equation of daylight of 3
Taurus is 9;28, and the equation of daylight of Gemini
is 11;12,/ and this is the compound (equation of 4
daylight).

However, as for the [differences][2] of the 5
equation of daylight, of Taurus alone (it) is 4;15,
and the equation of the/ daylight of Gemini alone 6
is 1;44. The ascension of Aries in this latitude
will be/ 22;40, and the ascension of Taurus 25;39, 7
and the ascension of Gemini 30;29.

Once these quantities are agreed upon we (go 8
on to) say that they have a zīj called the Khaṇ-
ḍakhādyaka/ of the works of Brahmagupta, and it is 9
that which is known in our countries as the *Arkand*
Zīj./ It includes, concerning the equation of day- 10
light, which they call [*caradala*][3] that we multiply
the equatorial shadow,/ it being called [*visuvac-* 11
chāyā][4], by a hundred and fifty-nine, and we divide
what results by/ sixteen, and there come out *fala*, 12
(Sanskit *pala*),each ten of which is a *sas*, and that
is the equation of daylight/ of Aries....[5] Then we 13
multiply this shadow also by ten and we divide the
result by three,/ and there comes out the equation 14
of daylight of Gemini, and thus if we compute it with
the assumed shadow/ there comes out the equation of 15
daylight of Aries as 5;19 and its ascension 22;34,
and the equation of/ daylight of Taurus [4];21[6], 16
and its ascension 25;33, and the equation of day-
light of Gemini/ 1;47, and its ascension 30;26. 17

The operation in the Indian copy is analogous 18
to this in that they mention in it/ the name *pala*, 19
this amount being among their weights of commodi-
ties, and its [weight][7] is/ fifteen *dirhams*,but 134:1
it occurs in their astronomical texts along with

[1]Text بج ; read ء .
[2]Text تفصيل ; read تفاضل .
[3]Text حردل ; read جردل as in the MS.
[4]Text يشق لحاى ; MS سق بجاى ; read بشوبجاى .
[5]Here a clause has been omitted from the MS.
[6]Text ج ; read د .
[7]Text وزائه ; read وزانه as in the MS.

the equation, whereas *sas*/ is *degree* in Persian. 134:2
It is their custom to use its parts as minutes/ of 3
the day, and its seconds, but not time-degrees. For
this reason there was put in the Indian copy/ the 4
ascensions for those (units), they being, for the
terrestrial equator, for Aries [4,38][1] (*pala*), and
for Taurus [4,59][2] (*pala*),/ and for Gemini [5,23][3] 5
(*pala*). If these numbers are multiplied by six
[minutes][4] there results for Aries/ 27;48 and for 6
Taurus 29;[54][5], and for Gemini 32;[18][6], and those
time-degrees/ are their ascensions in *sphera recta*, 7
approximately.

There is found from the Persian increments 8
(of daylight), in some of the copies (of manuscripts)
that the equatorial shadow,/ whenever it is five 9
digits, then operate with it as is mentioned in the
Arkand,/ but when it differs from it, then take for 10
each digit of the excess eight minutes of an hour,/
and subtract it from what comes out for the ascen- 11
sion if the excess is that of the shadow over the
five/ digits, but add it to it if the excess is that 12
of the five digits. This excess in/ our example is 13
[0];21[7], and its argument in hours is [0];2,48[7] if
for each/ digit and a half (we take) a fifth of an 14
hour, and its time degrees are [0;0],42[7]. If we
subtract it from/ the ascension which resulted for 15
us, it will decrease by one minute approximately.
This (following) operation is found/ in some of 16
the books, to multiply the equatorial shadow by
15[9][8], and divide/ the result by 16, and subtract 17
what comes out from 278, and divide the remainder
by ten/, and there comes out the ascension for 18
Aries. Then multiply the shadow by 65, and divide

---

1 Text بع ; read دلح.
2 Text رمط ; read دنط.
3 Missing in the text and MS.
4 Missing in the text.
5 Text يد ; read ند.
6 Text لح ; read ع.
7 Text ه ; read ح as in the MS.
8 Text ا ; read ٩.

the result by 8/ and subtract what results from 134:19
299, and divide the result by 10, and there comes out
the ascension/ for Taurus. (Next) multiply this 135:1
shadow by 10, and divide the result by 3, and sub-
tract/ what comes out from 323, and divide the re- 2
mainder by 10, and what comes out is the ascen-
sion// of Gemini./ This is precisely the first f.221b
operation (except that) in it the excess is sub-
tracted from the right ascension,/ then the re- 4
mainder is transformed from seconds of days into
minutes of time-degrees.

The first (operation) itself is found in 5
the *Shahriyārān Zīj*. In it the parts of the divi-
sion are made/ 160, 30, (and) 80, and they are ten 6
times the first so that the/ result is transformed 7
into time-degrees.

I encountered it in some of the commentar- 8
ies in another form. It is to multiply/ the shadow 9
of Aries by 114, and divide the result by 105, and
there comes out the excess of Aries, but for Taurus/
multiply by 13, and divide the result by sixteen, 10
and there comes out its excess, whereas for Gemini
divide/ by three. If we take this into considera- 11
tion in our example there would come out for the
ascension of Aries 22;4,/ and Taurus 25;[33],[1] and 12
Gemini 30;26.

It may be that with the two numbers of Taurus 13
one hundred is found, whereupon its ascension would
come out as/ 27;41. But the first is closer. And 14
among the marvellous things is (the fact) that some
foolish people attach to it an explanation/ of some- 15
thing not in the original, such as its saying to
multiply by 114 and divide by/ 150, because the 16
first is the diameter of the heaven, and the second
the total sine. If we make use of this/ reasoning 17
of his, oh what a fool!, accepting it from it (the
text) the excess for Aries would come out in our
example as/ [5;48,33][2], but if we computed by the 18
hundred and fifty minutes, the excess for Aries
would be// 4;3,[58][3], and its ascension 23;49.

---

1 Text كط ; read لج.
2 Text لح ح ر ; read لح ح ه.
3 Text لح ; read خ.

It is said also in some of the commen- 136:1
taries that if you have the excess for Aries, then
multiply it by/ nine and divide by eleven, and there 2
will result the excess for Taurus. Multiply it
also by/ four and divide it by eleven, and there 3
will result the excess for Gemini. If we operate
thus/ for the excess of Aries in our example, which 4
is 5;[19][1], the excess for Taurus will come out as
[4];21[2],/ and the excess for Gemini 1;[56][3]. [Va- 5
teśvara][4] prescribes in his zīj known as the [*Kara-
ṇasāra*/ to][5] multiply the equatorial shadow, for 6
Aries by ten, and for Taurus by eight, and for
Gemini/ by three and a third, and so the excess 7
of Aries in it for our example will be 5;[21][6],
and for Taurus/ 4;[16,48][7], and for Gemini, 1,47. 8
But [Vijayanandin][8] prescribes in the *Karaṇatilaka*,
which is "The [Best][9] of the/ Zījes", that the 9
equatorial shadow, for Aries be multiplied by
twenty, and for Taurus by sixteen,/ and for Gemi- 10
ni by seven, and so there will result the excess
of daylight in *ghaṭīs*.

In our example there will result (for) half 11
of the excess of daylight after transformation into
time-degrees,/ Aries 5;[21][6], Taurus 4;[16],48[10], 12
and Gemini [1];52,21[11]. It is like what/ preceded, 13
except as to the number for Gemini, for its frac-
tion is greater by a sixth of a part.

It is reported of *Yalṭabān* (?) the Indian, 14
to whom is attributed the well-known operation/
for the extraction of the chords of the circle, 15
that one should set the equatorial shadow in three
places,/ and in the first of them, subtract one 16
(minute) out of each one hundred and sixty minutes,

---

[1] Text لط ; read مط as in the MS.
[2] Text ج ; read د .
[3] Text مو ; read نو .
[4] Text يتشفر ; MS بسفر ; read بتشفر .
[5] Text بكون سارنضربه ; read بكرن سارنضربه as in the MS.
[6] Text ر ; read كا as in the MS.
[7] Text لولح ; read وح .
[8] Text بحيابند ; MS بحيانند ; read بجيانند .
[9] Text عدة ; read غرة as in the MS.
[10] Text لو ; read و as in the MS.
[11] ا missing in the text; present in the MS.

there remaining the excess for Aries./ In the 136:17
second of them, subtract three minutes out of
each ten minutes, there remaining the excess for
Taurus./ In third of them subtract three, there 18
remaining the excess for Gemini. According to this
in/ our example the excess for Aries will be [5;19][1] 19
and the excess for Taurus 3;45, and the excess for
Gemini/ 1;47. And because the excess for Taurus 137:1
differs from what preceded, he subtracts out of
each ten/ minutes two minutes. There results as 2
the excess for Taurus 4;17. And there is found
also in this connection/ that the (equatorial) 3
shadow at noon is increased by a third and a fifth
of a third of the equatorial shadow, and/ there 4
results the base. Then this base is subtracted
from [thirty][2], leaving the ascension for Aries,
and it is increased by/ one fifth the double of 5
the base, giving the ascension for Taurus, and
this fifth is added to the ascension for/ Taurus, 6
giving the ascension for Gemini.

I suppose that what is intended by the 7
extraction of the base is to add to the noon equa-
torial shadow/ a third of it and a fifth of its 8
third, since there is no place for the noon shad-
ow on the other days/ as far as this is concerned, 9
otherwise there would be for each day a new ascen-
sion./ // If the intention is what we have f.222a
supposed, then the base would come out in our
example as 8;12,12,/ and the ascension of Aries 11
[21];47,48,[3] and the ascension of Taurus 25;4,[41][4],/
and the ascension of Gemini 28;[21,34][5]. 12

There is found in some of the books of the 13
Persians another operation, which is this. It is
said, "Subtract/ the equatorial shadow from the 14
right ascensions for Aries, but increase by it for
Virgo./ Then subtract from the equatorial shadow 15
two digits and a half and a third, and subtract
what remains/ from the right ascensions for Taurus 16
and increase by it for Leo. Then subtract from

---

[1] Text وبط ; read هط as in the MS.
[2] Text ثلثين ; read ثلاثين .
[3] Text ز ; read كا as in the MS.
[4] Text بط ; MS لط ; read ما .
[5] Text زل ; read كالد ; MS كال .

the equatorial shadow/ three digits, and subtract 137:17
the remainder from the right ascension for Gemini,/
but increase by it for Cancer, and there will result 18
the ascensions of these signs for the locality in
question".

In our example we will obtain for the excess 19
of Aries 5;[21][1], and for its ascension 22;[32][2],
and for the excess for/ Taurus 2;31, and for its 138:1
ascension 27;23, whereas for Gemini the excess is
2;21,/ and its ascension 29;[52][3]. All of this is 2
remote from what is desired.

The author of the operation said, "As for the 3
people of Babylon, they multiplied the equatorial
shadow/ by twenty-five, and divided the result by 4
eighteen, and subtracted what results/ from thirty. 5
There remained the ascension for Aries. Then they
subtracted twice the ascension for Aries from sixty,/
and divided the remainder by five. There resulted 6
the base of increase for each sign, and they did/
add it to the ascension for Aries to (obtain) Taurus, 7
and to the ascension for Taurus to (obtain) Gemini,
and so on until/ Virgo". 8

If we follow this as in our example, the sub- 9
traction from thirty would be/ 7;25,50, and the ascen-
sion for Aries 22;[34,10][4], and the base of increase 10
2;[58],20[5],/ and the ascension for Taurus [25;32],30[6], 11
and the ascension for Gemini 28;30,50, and the ascen-
sion for/ Cancer 31;29,10, and the ascension for Leo 12
[34];27,30[7], and the ascension for Virgo/ 37;25,50, 13
but what we have related is more than enough.

---

[1] Text لط ; read كا as in the MS.
[2] Text ل ; read لب.
[3] Text يب ; read نب.
[4] Text لده ; read لدي as in the MS
[5] Text خ ; read غ.
[6] Text كعل ; read كهلب.
[7] Text لز ; read لد.

THE TWENTY-THIRD (CHAPTER) 138:14

ON THE DETERMINATION OF WHAT IS PAST 15

AND WHAT REMAINS OF DAY(LIGHT)

BY (USE OF) THE SHADOW

One may obtain what has passed of the day by 16
means of sines,/ either measuring by means of the 17
shadow or with the altitude. If you extract the
sines (you) dispense with/ shadows, and hence we do 18
not intend to include this topic (i.e. sines) here,
since our objective here is what can be obtained by/
the use of shadow(s), accurately or approximately. 19

The Indians have something to say in this 139:1
respect which is sometimes inclined toward deep in-
vestigation, but sometimes deviating from it./ Of
this (latter) type are things based on unjustified 2
assumptions, and it suffices, in/ examining them, to 3
criticize these assumptions. The reader of this
treatise need not think that we are unaware of such
cases,/ even though we do not relate them. More- 4
over, one need not doubt as to our opinion of it,
nor believe in its/ validity, in the event that he 5
is one of the mass of those who overestimate Indian
achievements without/ examining them. 6

Of that (sort of thing) is what I found in the 7
Pauliśasiddhānta, and the majority of them/ used it, 8
always adding twelve to the digits difference between
the two shadows, (that at the)/ time of the measure- 9
ment and (that at) noon, and dividing the arc of day-
(light), I mean its amount in minutes of the/ day 10
multiplied by six, by what resulted. They claim
that the result/ of the division will be what is 11
past of the day in minutes of the day if the measure-
ment is/ eastward before noon, while (the quotient 12
will be) the remainder of it if the measurement is
westward after noon./ That is because they should 13

be treated alike, and hence we will introduce/ 139:13
one of the two of them, and it is what passed, be- 14
cause it passed in actuality while the rest is po-
tentially passing. When the measurement is/ at 15
noon, then there is no [difference][1] between the two
shadows; the division is by twelve/ alone. Multi- 16
plication of minutes of the day for the whole day by
six is for transforming it into time-degrees./ Had 17
he divided by two, the time of half the arc of day-
light would have been obtained, and that is the/
arc of revolution for the time in question. But he 18
needed it in minutes of the day, not in time-degrees.
So we should/ divide it by six after the division 19
by two. For combining the two division (operations)
he divided by/ the product of the two// times f.222b
six so that there will come out for him the *gha-* 140:1
ṭīs of half the day(light)./ They will be equal 2
to the parts of the unequal hours, because each of
them results from the/ division of the time-degrees 3
of the arc of day(light) by six. The twelve is the
product of two/ times six, not the (number of) 4
digits of the gnomon, as some of them thought. And
so he prescribed increase of the gnomon by/ the 5
shadow at the time, and the subtracting of the noon
shadow from what results.

Since its [calculation][2] at noon is known, 6
it is also known that [one adds][3] to each [dif-
ference][4] between/ the two shadows dividing it, as 7
well as adding it in the case when the difference
vanishes, to transform the time-degrees into minutes
of/ the day. And when he found the shadows before 8
noon, (they are) shortening (and) lessening, and the
time/ passed from the first of the day for it is in- 9
creasing, he used between the two of them the ratio
in equivalence, which/ means in their language re- 10
cession, which implies that the ratio of the arc of
revolution of the heaven,/ i.e. of the celestial 11
equator, to the time of half the arc of daylight, is

[1]Text الفصل ; read الفضل .
[2]Text جيبها ; MS حسبها ; read حسبُها .
[3]Text الحمت ; MS illegible.
[4]Text فصل ; read فضل .

as the ratio of the noon shadow/ to the shadow 140:12
at the time. But since it is possible for the
noon shadow to vanish,/ the shadows themselves 13
were not used, but rather their differences,
since they also decrease/ with increase of the 14
arc of revolution, having the extreme at noon and
decreasing to a smallest for the day./ Since for 15
noon he divided the time-degrees of the arc of
daylight by the sum of twelve and/ the vanishing 16
difference, he also divided it here by the sum of
twelve and the non-vanishing/ difference. We 17
said that the western side of the gnomon is analogous
to the eastern side,/ and hence we dispense with the 18
explanation of one of the two.

[Brahmagupta][1] said, in the thirteenth trea- 19
tise of the ***Brahmasiddhānta***:/ "[Divide][2] a gnomon 141:1
as we may desire, and measure the shadow by it, and
add to it one of its units,/ and divide the result 2
into the minutes of the amount of half the day(light).
There result the minutes passed (since sunrise)/ or 3
remaining (until sunset). Conversely, divide the
minutes of half the day by the minutes passed,/ or 4
the remaining ones, and subtract from what comes out
one, and the shadow remains".

But I think that the translator marred the 5
expression, and that the purpose of the increased one/
or the [decreased one][3] was the gnomon itself, cor- 6
responding to the twelve (in) the preceding (rule).

Then Pṛthūdakasvāmin (text: Brtuswām) said 7
that this is wrong, which is affirmed by what I
found, but I do not/ know what led [Brahmagupta][1] 8
to it, since it is not valid anywhere, either on the
terrestrial/ equator, or in (places) other than it. 9
And perhaps he needed it for a purpose I do not know.
Such/ operations are corrupt, or made for simplifica- 10
tion and approximation.

I found in books translated from the Indian 11
languages/ in the first days of the ʿAbbāsid dynasty, 12
Indian names appearing in them without being trans-

[1]Text برهمكويت ; read برهمكوبت .
[2]Text جبر ; MS جزّ .
[3]Text المنقوس ; read المنقوص as in the MS.

lated/ or without carrying over their meanings 141:13
into Arabic, and this is it:

"Measure the shadow of the gnomon at the 14
time and add to it twelve always,/ and subtract the 15
noon shadow from what results. Then multiply the
ghūlijāt (Sanskrit *ghaṭikās*) of half a day/ (for) 16
your day by six always, and divide the result by
what remains to you, and what results, double it./
Take a [fifth of it][1], and it will be the hours 17
passed before noon, or (what) remain after it"./
As for the *ghūlijāt*, it is an expression for the 18
minutes of the day in one of their languages, or/
after having been incorporated (into another lan- 19
guage), but we have never happened to hear it. And
this operation up to the doubling of the quotient
resulting (is)/ in agreement with the first op- 142:1
eration, and the subdivision of both is the *ghaṭī*
and each two and a half/ of them is an [equal][2] 2
hour . . . . . . (a hiatus). To [change][3] *ghaṭīs*
to hours take a/ fifth of its double. From this 3
you see that the inverse of that is that if we
wanted to transform hours/ into minutes of the day, 4
the hour being two (day-)minutes and a half, so we
put it down in/ two places. Then we double one 5
of the two and halve the other, and we add the re-
sult of the two. It will be/ the desired (thing). 6

Because the Indian zījes are composed in 7
verse form called by them *śloka*, thus some of the
partisans of the Sindhind zījes/ composed their 8
zīj analogously (in verse),// and they said with f.223a
regard to this.

If it be thy pleasure to determine the hour of 9
the day,
Then take a stick by which [thou livest][4], 10
Which is the deed of a wise man
Who would investigate the seas (of knowledge) 11
Rich and full.
So, let thy stick be, mark well, 12
Its length ten digits.

---

[1] Text خمسة ; read خُمسهُ .
[2] Text منسوبة ; read مستوية .
[3] Text فقيل ; read فنقل .
[4] Text بعشر ; MS بعسه ; read تعسه .

Then add[1] [two][2] to the ten 142:13
This will be useful to thee.
And if thou obtainest not a good result, then 14
try again.
So, set up the stick, and take the measure 15
of its shadow in the sun.
Then add to it the equal of the stick's amount 16
to all that.
What leads(?)[3] one to the good is not as plain 17
as that which misleads him.
Then [we cast off][4] from it the amount of the 18
shadow at noon.
Set apart the remainder to use it judiciously. 19
Then put a *ba'* (two) and an *'ayn* (seventy) 143:1
Without fearing disgrace.
[Then][5] divide it by what [thou hast set aside][6] 2
previously.
Then compute what thou findest, and compute what 3
remains.
Thus is the procedure in this whose base thou 4
seekest.
And if thou art still facing day, then that is 5
the number (?)[7]
Of hours that passed and left the earth as a 6
measure.
Whereas if thy day is about to finish, then it 7
is the remaining part[8].

Thus Muḥammad b. Ibrāhīm al-Fazārī put it in 8
his astronomical ode. And he said in it concerning 9
the time passed of the day:

So if thou seekest what has passed and what
remains

---

[1] Text زد ; MS زيد .
[2] Text ثنتين ; read اثنين as in the MS.
[3] Text هادى ; MS عادى .
[4] Text تلقى ; MS نلقى .
[5] Text ثمّ ; read ثم .
[6] Text القيت ; MS اتفتت .
[7] Text قدرا ; MS صدرا .
[8] Text فتوليهن ; MS فتوقيهن .

Of the day by reliable computation, 143:10
Then do it slowly, may God guide thee. 11

(Make) a stick whose [graduations][1], for elegant measurement[2] 12
Are six and six (i.e., twelve) and let patience support you,
Its length being of the amount of a span. 13

So set it up in a level place. 14
Then look at the shadow where it ends.
And measure it with the stick. 15

What results [ending][3] at the numbering 16
And the computation is as thy shadow at the time.
So add to it the like of the stick's shadow, 17

And subtract from it the shadow at noon of thy day. 18
Allot[4] that, all of it, with persistence[5].
In that is the perfection of the affair. 144:1

What is left, divide by it here, 2
As two with seventy until it is finished.
This, upon my soul, is plain in meaning. 3

Understand, if thou dividest what results, 4
And those are the hours obtained in a reliable way.
From the straight, the well-traced computation. 5

It is, if the day is facing you, 6
What passes by and by
Until it passes the half (i.e., noon) fully[6] (and) completely. 7

But it is, if the day is retreating, 8
What remains finally[7]
Until the setting of the sun so that it is not seen. 9

---

[1] Text قدره ; MS جزه.
[2] Text القدر ; MS القدرة.
[3] MS فانتهى ; missing in the text.
[4] Text واحص ; MS واخضرى.
[5] Text كله بهما ; MS كل فصمكا.
[6] لا missing in the MS.
[7] Text آخرا فآخر ; MS آخرا فاخرا.

This is an example of the majority of 144:10 these verses, which add twelve to the shadow always,/ and subtract from the result the noon shadow, 11 and divide the remainder into/ seventy-two as 12 an invariable procedure to obtain the hours passed before noon/ from the beginning of the day, or re- 13 maining after it until the last of the day(light), and its relative[1] magnitude.

So, let the gnomon be *AB* (in Figure 42) 14 and *B*[*H*][2] equal to it, and it is evident/ that if 15 we add to the noon [shadow][3] twelve and we subtract from the result the/ noon shadow, then[4] the remain- 16 der will always be *B*[*H*][2]. And we suppose that *B*[*G*][5] is/ six. So the area (formed by) *B*[*G*][5] (and) [*D*]*G*[6] 17 will be seventy-two. Had we not added/ the twelve, 18 nothing would have been left over, and then the division at noon would have been (by)/ nothing. But 19 the one performing the operation should/ obtain 145:1 six at that time, because his object was the (determination of the) unequal hours. So he made/ the division by the amount of the constant gnomon 2 length whether we have a shadow or not. And when/ the seventy-two is divided by *HB* there comes out 3 *B*[*G*][5], the six. If/ his operation is pursued at 4 noon by subtracting the noon shadow from the shadow at the time/ and increasing the stick('s length) 5 by the remainder, even though it vanishes, and dividing seventy-two/ by it, then its parts at all 6 times of the day will be similarly (obtained.) So, at (times) other than/ noon, let the shadow of the 7 gnomon be *BZ*, and the noon shadow *BT*./ The dif- 8 ference between them is *ZT* and we cut off [*B*]*K*[7] equal to *ZT*./ *K*[*H*][8] will be the divisor, to which 9 we add the area (formed by) *K*[*H*][8] (and) [*K*]*M*[9] equal/

---

[1] Text نسبته ; MS لنسبته.
[2] Text ج ; read ح.
[3] Missing in the text.
[4] Text اذن ; MS اذ.
[5] Text د ; read ج.
[6] Text ه ; read د.
[7] Text ج ; read ب.
[8] Text ب ; read ح.
[9] Text ح ; read ك.

Figure 42

to the area of *B*[*G*][1] (and) [*D*]*G*[2]. And so *B*[*E*][3] 145:10
will be the number of the hours for the difference
ZT.

There is found in some of the copies division 11
by seventy-two// instead of/ dividing it by f. 223b
what we mentioned. But that is a mistake on the
part of the copyists, (and is)/ indeed [distant][4] 13
from the objective.

[1]Text د ; read ج .
[2]Text ه ; read د .
[3]Text ح ; read ه .
[4]Text مبعد ; MS يبعد .

In some of them it is explained that if 146:1
the shadow is greater than seventy-two, then mul-
tiply/ sixty by seventy-two and divide the result 2
by what is found in parts of the stick./ And the 3
difference between these operations will suffice to
convince the judicious (individual) of the disag-
reement among their results/ (even) for the same 4
example, especially if one compares by [rigorous][1]
computation.

Here the printed text has omitted a passage, which, however, appears in 158:10 - 160:4. This passage has been restored to its proper place below.

Of what resembles this is an operation, 158:10
the author of which is unknown, suggested by intui-
tion and common sense./ It is to subtract the noon 11
shadow present at the time, and from what remains
(subtract) the/ twelve parts of the gnomon. And 12
if after that there remains sixty, an hour has
passed, and if/ there remains twenty-four, two hours, 13
and if there remains twelve, three hours, and if/
there remains two and fifty (minutes?), four 159:1
hours, and if there does not remain anything ex-
cept the noon shadow and the/ stick's shadow it 2
is five hours.

For the Indians there is another operation 3
also, they memorize it in verses and by it they
extract,/ instead of the unequal hours (the) *mu-* 4
hūrta, and it is a part of fifteen of/ the day or 5
of the night, and we have put what is in their poem
in this table:

[1]Text الخفى ; read الحقى as in the MS.

(*Muhūrtas*) passed before noon	1	2	3	4	5	6	7	
Increase of the shadow beyond the noon (shadow)	96	60	12	6	5	3	2	[0][1]
(*Muhūrtas*) passed after noon	[14][2]	13	12	11	10	9	[8][3]	

Figure 46

When (they) subtract from the *muhūrta* its fifth   159:6
it will become unequal hours, whereas/ if they add to   7
the unequal hours a quarter of them they will be transformed into *muhūrtas*.
I read in the zīj called the Hārūnī that if the   8
hypotenuse of the noon/ shadow is multiplied by a hundred and fifty and the result divided by the hypotenuse   9
of the shadow at the time of the measurement, and the
arc sine of/ what comes out is found in *kardajāt* of the   10
sine, and for each *kardaja* an hour is taken, (the result) will be/ the hours passed, or remaining.   11
Let, for it, *AG* (in Figure 43) be the meridian   12
line and *E* is the center of the horizon/ and [*F*]*OT*[4]   13

Figure 43

[1]Text ة ; read ق as in the MS
[2]Text لا ; read ر as in the MS.
[3]Text ط ; read ع .
[4]Text لعط ; MS بعط ; read فعط . In the figure, for س read ص, as in the MS.

the triangle of the day, and *LSH* the triangle   160:1
of the time, and *TS*/ is of the common part between
the planes of the parallel circle and the horizon,
and let the head of the gnomon be/ *E* on the horizon,   2
and its length from the erect (?) line (is) *EC*, and
both/ [*E*]*Y*[1] (and) [*E*]*K*[1] are hypotenuse(s) of the   3
noon shadow, and it (has been shown in what preceded that the ratio of the gnomon/ to the hypotenuse   4
of the shadow is the ratio of the sine of

Here the displaced passage ends.

[the][2] altitude/ to the total sine. And the   146:5
gnomon is made a mean proportional in the ratio between *EY* (and) *KE*, the hypotenuses of/ the two shadows.   6
So the ratio of *EY* to *EC* will be as the ratio
of *OE* to/ *OF*. And the ratio of *EC* to *EK* will be as   7
the ratio of *LH* to *HE*,/ which is equal to *OE*. By   8
the equality in the disturbed ratio, the ratio of
EY to/ *EK* is as the ratio of *LH* to *OF*. But the   9
ratio of *LH*/ to *OF* is as the ratio of *HS* to *OT*. So   10
the ratio of *EK*/ to *EY*, hence is as the ratio of   11
OT to *HS*. And so when the/ hypotenuse of the noon   12
shadow is multiplied by the versed sine of the day,
I mean the versed sine of half the day,/ and the   13
result is divided by the hypotenuse of the shadow
at the time, there comes out the compound (*HS*) of
the arc of revolution/ which, if it is subtracted   14
from the versed sine of the day, there will remain
the versed sine of the arc of revolution between
the time of the measurement/ and noon. And that   15
is *HS*. But *HS* is not/ the sine of a certain arc   16
even though there may exist an arc of the day-circle/ whose sine is line *HS*. It is compounded of   17
the sines of two arcs (which are)/ contiguous.   18.
For establishing how to determine it, we   19
extend perpendicular *ED* to *OT*,/ and we extend   147:1
DM parallel to *HS*, and *D* will be the center of/
the day circle, and *MS* (is) the sine of the equation of daylight in the small circle,// and *MH*   2 f.224a
will be the sine of the/ arc of revolution in it,   3
after the revolution of the equation of daylight.

[1]Text ص ; read ه .
[2]Text ارتفاع ; MS الارتفاع .

But the originator of this operation/ set up *OT*, 147:4
as having the place of the total sine, and he assumed
for it the hundred and fifty. There comes out/ *HS*
according to it. And behold the setting of the 5
total sine is not in hours, because of the fact
that it is the/ sine of ninety parts, *SH* will be 6
the sine of the arc of revolution in hours/ passed. 7
But this is not valid for locations having (non-
zero) latitude, except at two times, the equinox(es)./
TS at that time will be *EB*, the perpendicular to *LS*, 8
and *TO*/ will be *OD*, and *HS* will be *HM*. Its validi- 9
ty will remain at the/ terrestrial equator because 10
the [centers][1] of the daily circles are (then) in
the horizon plane. And so the/ author of the opera- 11
tion includes the positions having latitude in it,
and he treats them as alike,/ or it is as though he 12
thought that *OD* (and) *HM* the two remaining (things)
from *TO* (and) *SH*/ after the deletion from them of *TD* 13
(and) *SM* which are equal, as though the two remain-
ders still have the ratio/ between [*LH*][2] and [*OF*][3], 14
the two which he explained.

Yaʿqūb b. Ṭāriq inclines in (the direction) 15
of similarity to it in his saying: "Divide by the
hypotenuse of/ the shadow at the time a thousand 16
and eight hundred, and multiply what comes out by
a hundred and fifty./ And divide what results by 17
the sine of the noon altitude, and there comes out a
sine. We find/ its (corresponding) arc and take of 18
it for each fifteen degrees one equal hour".

That is because the thousand and eight 148:1
hundred is the product of the gnomon and/ the to- 2
tal sine. And the ratio of *KE*, the hypotenuse of
the shadow for the time, to *EC*, the gnomon,/ is as 3
the ratio of *EH*, the total sine, to *HL*, the sine of
the altitude at/ the time, and it is the quotient; 4
and its ratio to *HS*, the arrangement (?) of/ the 5
arc of revolution, is as the ratio of *OF*, the sine
of the noon altitude, to *OT*,/ the day sine. But 6
when he took *OT* as the total sine, the state of
affairs will be/ as was mentioned. But because 7

---

[1] Text مركز ; read مراكز as in the MS.
[2] Text عح ; read لح.
[3] Text عس ; read عف.

the factor in both[1] of the ratios is the/ total 148:8
sine, it would have been better to hold off, and to
multiply the gnomon by the square of the total sine./
And then there would result two hundred and seventy 9
thousand, and we divide it always by the product of
the divisor/ by one of the two ratios in the divi- 10
sor by the other, I mean the product of the/ hypot-
enuse of the shadow and the sine of the noon al- 11
titude, in order that he get from it/ what was
obtained previously.

In the Shāh Zīj, for ascertaining the (part 13
of the day) passed he directs division by the sine
of/ the altitude at the time, of a thousand and 149:1
eight hundred. There comes out the hypotenuse of
the shadow for that time,/ and by it he divides 2
the product of the [length][2] of the computed sine
(i.e., the day sine) and the hypotenuse of the noon
shadow./ What comes out is subtracted from the 3
length of the computed day sine, and the remainder
he subtracts from a hundred/ and fifty, and the 4
arc sine of the remainder is found. And so it
will be the equation of the sine. If the alti-
tude is/ easterly, subtract the equation of the 5
sine from ninety, and if the altitude is westerly/
increase by it (the) ninety, and there results the 6
arc of revolution of the sky.

In the last parts of the operation a mixup 7
has occurred because of ignorance in the craft
(of astronomy), and that is/ that the product of 8
the gnomon by the total sine, if it is divided by
HL (in Figure 43) there comes out/ *EK*, but if it 9
were divided by *EK* there would come out *HL*. The
length of the/ computed sine is *OT*, the day sine, 10
and that which comes out for him is *HS*,/ which he 11
subtracted from the day sine. The operation up to
this place is rigorous, and verily/ there comes out 12
for him the versed sine of the arc of revolution
between the time (in question) and noon, and it is
a/ versed sine; if its arc is found and subtracted 13
from half the arc of daylight for the/ eastern al- 14
titude, or added to it for the western altitude// f.224b
there results the arc of revolution for the (time)

---

[1] Text كلا ; MS كلى.
[2] Text ظل ; read طول.

passed from the first of/ the day. But when the 149:15
finding of the arc versed sine is continued for the
direct sines [on both sides][1], and if/ it is less 16
than the total sine he subtracts it from a hundred
and fifty and finds the arc sine of the remainder.
It is that/ which he calls the equation of the sine, 17
and he subtracts it from ninety, and if the versed
sine is more/ than the total sine, subtract from 18
it a hundred and fifty and find the arc (function)
of the remainder, and he adds the equation of/ the 19
sine to ninety and there results the arc of the
versed sine. Then he takes into consideration the
direction/ of the altitude to be increased over 150:1
half the arc of the day, or decreased. Thus he
knows/ what was dropped from the construction, and 2
what is curtailed from it should be subtracted and
what comes out should be subtracted/ from the length 3
of the computed sine. Then he takes the difference
between what remains and the total sine./ It is the 4
equation of the sine; he finds its (corresponding)
arc, and if the excess is to the total sine, he
subtracts/ the arc of the equation of the sine from 5
ninety. But if the excess is to what remains, the
arc of the/ equation of the sine is added to ninety, 6
and what results after the addition or subtraction,
one considers/ the altitude, and if it is easterly, 7
subtract it from half the arc of daylight, but if
it is/ westerly add it to it, and there results the 8
passed arc of revolution.

The Khaṇḍakhādyaka Zīj registers this opera- 9
tion in full accuracy./ For its author says: "Mul- 10
tiply the day sine by the hypotenuse of the noon
shadow and divide/ what results by the hypotenuse 11
of the shadow at the time, and what comes out is
the equation. Subtract it from the day sine,/ and 12
make what remains an arc versed sine. And divide
it by six, and there results what remains of/ min- 13
utes of the day until noon, or what has passed of
it". The indications of its validity are evident/
from what has preceded. 14

Then he said: "And when you have dropped the 15
equation from the day sine, and the remainder is/

[1] Text نظرفان ; MS بطرفان .

more than the total sine, then subtract from it 150:16
the total sine, and make what remains an arc (sine)./
Add to its minutes five thousand and four hundred 17
minutes, and there results (the amount) from/ the
time (in question) until noon". This will be the 18
time by which what passed of the day, or what/ re-
mained of it, lags the equation of daylight, because 19
the equation which comes out is *HS* (in Figure 43),/
and if it is less than *MS* the difference between 151:1
it and *TO* would be greater/ than *OD*. And if there 2
is subtracted from it *OD*, the total sine, in the
parallel circle, the remainder,/ which is a line from 3
D less than *TD*, is the sine of an arc/ whose begin- 4
ning is from diameter *DM* in a direction contrary to
O, I mean in the direction of the horizon./ And so, 5
if he adds it to the quadrant which is from diameter
DM toward *O*, and whose minutes are/ as explained, he 6
obtains the desired arc.

He should have said for completing the divi- 7
sion (into special cases, that) when you subtract/
the equation from the day sine and nothing remains, 8
the (time) passed or remaining is equal to the/ equa- 9
tion of daylight.

Then he said: "If you want to, subtract the 10
sine of the equation of daylight from the equation/
if the solar declination is north, and add it if it 11
is south, and make the result/ an arc (sine), and 12
add to it the equation of daylight if the declination
is north and subtract it/ from it if the declination 13
is south. There results the (time) passed or re-
maining". And that, in our northern example (is
that)/ if *SM*, the sine of the equation of daylight, 14
is subtracted from *HS*, the equation,/ there remains 15
MH, and it is the sine of the arc. If it is added
to the equation of daylight/ there results what is 16
between the rising point on the parallel circle and
H, and that is the (time) passed./ The southern 17
(case) is analogous, except for the increase. Then
he said: "If it is not possible to subtract the
equation of daylight/ from the equation, then make 18
the equation an arc versed sine, and it will be
the time passed".

But I think that this is ill expressed by the 19

translator, because such a thing will not evade/ 151:19
Brahmagupta. Indeed, one should find the corres- 152:1
ponding arc of the difference between them directly,
and subtract that/ arc from the equation of day- 2
light and there results the (time) passed or what
is remaining. However, [Vateśvara][1]/ prescribes the 3
multiplication of the day sine by the difference be-
tween the two hypotenuses of the shadow// for the f.225a
time and the/ noon shadow and the division of the 4
result by the hypotenuse of the shadow for the time
so that there will come out for him the versed sine/
of the arc of revolution from the time (in question) 5
to noon, and that is because it is the difference be-
tween the arrangement of/ the arc of revolution and 6
the day sine in these quadrants. (As for) operations
for the determination of the hypotenuse of the shad-
ow for the time by/ means of the (time) passed of 7
the day, they are based upon what we said. Of that
(sort is) what is in the [*Karaṇatilaka*][2],/ the "[Best][3]
of the Zījes", that one increases the sine of the 8
equation of daylight, (if) northerly, by the total
sine,/ and subtracts the sine of the equation of day- 9
light (if southerly), from the total sine, and there
remains the day sine./ And make the difference be- 10
tween the (time) passed of the day and half the arc
of daylight/ a versed sine, and subtract (it) from 11
the day sine and divide by what remains the product
of/ the day sine times the hypotenuse of the noon 12
shadow. And there comes out the hypotenuse of the
shadow for the time.

For he who knows about the inversion of opera- 13
tions, the switching of multiplication and division,/
one for the other, and addition and subtraction like- 14
wise, and the arc sine and [sine][4] (operations),/
those operations will not be hidden from him as being 15
the inverse of the preceding one.

In it (the Khaṇḍakhādyaka?) Brahmagupta added 16
that the arc of revolution between the time and noon,/

[1]Text يتشفر ; MS ببتيشفر ; read يتشفر .
[2]Text كون تلك ; read كرن تلك as in the MS.
[3]Text غرو ; read غرة as in the MS.
[4]Text التجيب ; read الجيب as in the MS.

if it is more than fifteen, i.e., a quarter of 152:17
sixty, then subtract it from/ thirty, i.e., half 18
of it, and make what remains a versed sine. And
subtract it from the double of the/ total sine. 19

Balabhadra the commentator said in this 153:1
respect: "Subtract from the arc of revolution fif-
teen (day-minutes),/ and make what remains a sine, 2
and add it to the total sine". And each of the two
of them give/ the versed sine of that arc of revolu- 3
tion. Of that (is) what is about it in the *Karaṇa-
sāra*, the "Annihilator of the Zījes",/ to subtract 4
the equation of daylight, (if) [northerly][1], from
the (time) passed, and increase it by the equation
of daylight (if)/ southerly, and make the result a 5
sine and increase by it the sine of the equation of
daylight (if) northerly,/ and subtract from it the 6
sine of the equation of daylight (if) southerly.
There results the part of the division. Divide/ it 7
into the product of the day sine and the hypotenuse
of the noon shadow. And there comes out the hypot-
enuse of the shadow/ for the time. 8

[1]Text الجنوبي ; read الشمالي .

THE TWENTY-FOURTH (CHAPTER) 153:9

ON THE AZIMUTH AND ITS ASCENSION 10

The altitude and the shadow and the azimuth 11
are functionally dependent at a single time so that/
each one of them is determined if any (one) is known. 12
Thus the magnitude of the shadow leads to the deter-
mination of the altitude,/ and this in turn deter- 13
mines the azimuth, because it is on the common part
(between) the planes of the horizon/ and the alti- 14
tude circle whose position on the horizon is deter-
mined (by) the amount of the azimuth, and just as/
the time of day becomes known by the altitude, like- 15
wise, it may be determined also/ from the azimuth. 16
Let it be known that *ABGD* (in Figure 44) is 17
the meridian circle and [*B*]*ED*[1] the eastern/half of 18
the horizon, for example, and *AEG* half the celestial
equator, with/ pole *T*. And let *O* be the position 19
of the sun at the time (in question) and its dec-
lination/ north, except in the fourth [picture][2] 154:1
where it is south, and let there pass through it
from *S*,/ the zenith, circle [*O*]*HZ*[3] of the circles 2
of altitude. Let its amount be/ *OF*[4], and so *EF* 3
will be its rising amplitude, and *CE* its equation of
daylight./ And let us describe about pole *Z* and at 4
a distance equal to the side of a square (inscribed
in a great circle) arc *MK*. Evidently it is to 5
the amount of the complement of angle *Z*. And so,
known to us are azimuth *EH*,/ and declination [*O*]*L*[5], 6
and (terrestrial) latitude *SA*, which is to the amount
of angle *E*. And because/ *EM* is the complement of 7
the azimuth *EH*, so the ratio of its sine to the sine
of *MK* is as the ratio of the/ sine of *ED*, the quad- 8
rant, to the sine of *DG*, the complement of the local

[1] Text ن ; read ب.
[2] Text الصوة ; read الصورة as in the MS.
[3] Text ه ; read ع.
[4] Text عف; MS عب (?).
[5] Text ه ; read ع as in the MS.

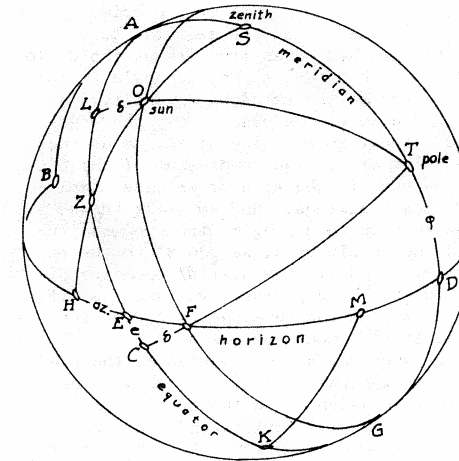



Figure 44

latitude. And so *MK*/ is known, and the ratio of 154:9
its cosine, I mean angle *Z*, to the sine of angle *E*,/
is as the ratio of the sine of *EH* to the sine of *ZH*. 10
And so *ZH*, called/ the mean altitude, is known. 11
And the ratio of the sine of *ZS*, its complement, to
the sine of/ *SA* is as the ratio of the sine of// f.225a
ZO, the equation of the altitude, to the sine of *OL*,
the/ declination of the sun. And the equation of 13
the altitude is known. So, if we add it to the
mean altitude,/ for southern declinations, there 14
results *OE*, the altitude of the sun. And when the/
azimuth vanishes, or the declination vanishes, the 15
mean altitude will become the modified (altitude)
itself. And verily/ the method of extracting the 16
altitude from the azimuth is thus (made) evident.
However, as for the determination of the 17

(time) passed of the day by means of the azimuth, 154:17
the ratio of the/ sine of *EH*, the azimuth, to the 18
sine of *EZ*, the retained (quantity), is as the ratio
of the sine of/ angle *Z* to the sine of angle *H*, the 19
perpendicular. And so the retained (quantity) is
known,/ and the ratio of the sine of *OZ*, the equa- 155:1
tion of the altitude, to the sine of *ZL*, is as the
ratio of the/ sine of *OT*, the complement of the dec- 2
lination of the sun, to the sine of *TS*, the comple-
ment of the/ local latitude. And so *ZL* is known, 3
and if the azimuth is in a single direction, as in/
the first and fourth pictures, we add *ZL* to the re- 4
tained (quantity), and if they are in/ opposing di- 5
rections, as in the third picture, we take the dif-
ference between *ZL* and the retained./ And so the 6
result will be *EL*, the mean ascension.

However, when the azimuth vanishes, the pre- 7
served (quantity) vanishes (also), and *ZL* will be the
mean/ ascension. However, when the declination van- 8
ishes, then *ZL* (also) will vanish, and the/ pre-
served will be the mean ascension. But the arc of 9
revolution is *CL* on the celestial/ equator [and *FO*][1] 10
on the parallel circle. And it is necessary to add
CE, the equation of daylight, for/ northern declina- 11
tions, to the mean ascension, and to subtract (it)
from it for southern declinations, as in the case of
the fourth/ picture, so that there results the arc 12
of revolution for the (time) passed in the case of
an eastern azimuth, or the arc of revolution/ for
the (time) remaining with a western azimuth. 13

However, with a vanishing declination, the 14
mean ascension is exactly the arc of revolution for a
vanishing/ equation of daylight, and that is what 15
we wanted to explain.

The azimuth of the shadow is equal to the 16
azimuth of the sun in amount and opposite it/ in 17
direction because the two are always along the hypot-
enuse in the directions of its ends, or in two/
opposite quadrants. 18

When it is [needed][2]// by means of the f. 226a
shadow and the altitude, then let/ *ABGD* (in Figure 156:2
45) be the circle of the horizon and *B* in it the

[1]Text وقح ; MS وحح ; read وفح.
[2]Text احيتج ; read احتج.

Figure 45

eastern direction, and *A* (gives) the direction 156:2
of/ south, and the diameter *HET* is the common part 3
between the planes of the horizon and the circle of/
altitude, and *H* on it is in the direction of the 157:1
sun. *BH* will be the distance of/ its azimuth from 2
the east to the south in our example. And the dis-
tance of its azimuth from/ the meridian line is *AH*. 3
Undoubtedly the extension of the shadow (is) along
diameter *HT*. And its end (is)/ from *E* in the direc- 4
tion of *T*. So *TD*, equal to *HB*, is the distance of
the azimuth of the shadow/ from the west in the di- 5
rection of north. And let us suppose that *EZ* is
equal to the cosine of the altitude of/ the sun, 6
known either by itself, or determined by means of
the shadow. We erect *ZK* perpendicular to the hori-
zon plane/ equal to the sine of the altitude. We 7
extend *ZM* perpendicular/ to [*F*]*O*[1], the common part 8

[1]Text بح ; read فح . In the text figure ح is missing; present in the MS.

between the planes of the horizon and the parallel 157:8
circle. And we connect *K* (to)/ *M*. *KM*[*Z*][1] will be 9
the triangle of the day, and the ratio of *KZ* in it,
it being the sine of/ the altitude, to the base 10
[*Z*]*M*[2], is as the ratio of the sine of angle *M*, which
is the amount of the/ complement of the latitude, to 11
the sine of angle *K*, which is the amount of the lat-
itude. And so *ZM* is known/ and *ZL*, the argument of 12
the azimuth, is known, because it is the sum of *ZM*
and *ML*, the sine of/ the rising amplitude, for south- 13
ern declinations, and the difference between them
for northern latitudes./ The azimuth *HB* is southern, 14
except when it is the excess of the sine of the/ ris-
ing amplitude over the base *ZM*. At that time it 15
will be northerly, and whenever the two are equal/
the azimuth will be along the east-west line. And 16
for the determination of its distance from the east-
west line,/ the ratio of the square of *EZ* to the 17
square of *ZL* will be as the ratio of the square of
EH to the/ square of *HY* because of the similarity of 18
the two triangles *EZL* (and) *EHY*. If we multiply the
argument of/ the azimuth by itself and the total sine 19
by itself, then we[3] take the sum of the two results
in/ the other, and we divide the result by the 158:1
product of the cosine of the altitude by itself and
we take the (square)/ root of what comes out from the 2
division, thus *HY* will be the sine of the distance
of the azimuth/ of the sun from the east. The dis- 3
tance of the azimuth of the shadow from the west will
equal it. And if we want the/ distance of the azi- 4
muth from the meridian line we multiply each one,
the argument of the azimuth/ and the cosine of the 5
altitude, by itself and subtract the lesser of the
two from the greater. Then/ we multiply what re- 6
mains by the product of the total sine by itself,
and we [divide the product by the square of the co-
sine of the altitude. We][4] take the root of what

[1] Text كم ; كمز as in the MS.
[2] Text دم ; read زم.
[3] Text اخذنا ; MS اخذ.
[4] Missing in the text.

results/ from the division. *HC* will be the sine 158:7
of the distance of the azimuth of the sun from/
the south in the direction of the east, and it 8
will equal the distance of the azimuth of the shad-
ow from the north in the direction of/ the west. 9
And that is what we wanted (to do).

The passage missing here,
158:10 - 160:4, is an intrusion
in the printed text. In this
translation it has been restored
to its proper place at 146:4.

THE TWENTY-FIFTH (CHAPTER) 160:5

ON THE RECITAL OF THE OPINIONS OF THE IMAMS REGARDING THE TIMES OF PRAYER AND WHAT IS RESORTED TO (6) IN DETERMINING THEM (7)

The sun is the chief sign for time determination. (8) Because the Ḥarrānians, and the Hindus, and the Magians,/ and all those who worship (*ʿaẓm*) (9) the luminaries take the times of their rising and setting and their culmination as/ times for prostrations and worship, (10) for their rising is their (time of) kindling, and their culmination/ is their (11) reaching of perfection, and their setting is their leave-taking. But all (these) are sects whose truth was not certified by Islam/ at any time in the past. Verily prayer at the three times (above-)mentioned (12) (is) forbidden us that we be distinguished from (13) them. It is said that the sun rises between the two horns of Satan,/ with the meaning that his as- (14) sociates worship the sun at that time.// So (f.226b) it is as though he raises it over them to seduce them,/ not what was said of him that Satan hinders (15) it from rising or setting until it burns him./ That is impossible and incomprehensible, and not (16) becoming to the affairs of the kingdom (of God). The horns/ are the edges, and they can be used in (17) connection with the sun. So one may say one of its horns has risen. Other/ nouns can be used in (18) connection with it. So one may speak of the sun's eye, face, or head. I think/ the calling of what (19) is between the shadow and the [sunlight][1] the resting-place of Satan is for a similar reason, since it resembles [what is between the day and the night. So staying in it (for prayer) was disapproved and forbidden, and the time of the first prayer was made to be][1] the declining of the sun from its (161:1) perfection (i.e., culmination). As for the two prayers which are at the end of the day and night,/ one of them is before its rising and the other is (2) after its setting at the two times of disappearance of its body./ The times of these two (prayers) are (3) not determined by it (the sun), because at those/ times its rays make a spreading [light][2] which is (4) a sign for people for/ the dawn prayer, and similarly (5) for the prayer of darkness, whose time is the vanishing of the twilight corresponding to/ the morning (6) in reason and in being. The prayer of setting is at the first (part) of the night. A prayer was not/ set for the first part of the day, as was mentioned, (7) because of the sunworshipers devoting themselves to prayer at that time./ It was exchanged (8) for the afternoon prayer in the other half of the day. So the two prayers, the dawn/ and the darkness (9) (prayers depend upon) the rays of the sun, while the setting (prayer depends upon) its body. And (finally) the noon and afternoon (prayers depend upon)/ the disappearance of its rays, I mean (10) the shadow, and it is the subject matter we are dealing with.

As for defining their times by means of the (11) remains (in the tradition, etc.), verily information about it came from the Prophet,/ upon him the (12) prayers of God and peace!, "Verily Gabriel came to me twice at the door of the Kaʿba and we prayed/ the noon (prayer) when the shadow is like the rope (13) of a trap (i.e. very thin), then the afternoon prayer when the/ shadow of anything is the equal (14) of it(self); then the sunset prayer when the sun falls and the fast is broken,/ then the prayer of (15) nightfall when the twilight disappears; then the morning prayer when dawn arises and/ food is for- (16) bidden for the one who fasts".

"On the second day he (i.e. Gabriel) prayed (17) with me the noon (prayer) when the shadow of each

---

[1] Text الصبح ; read الضح as in the MS.

[1] This passage in the MS was left out of the text: بابين النهار والليل فكره القعود فيه ونهى عنه وجعل وقت الصلوة

[2] Text الصبح ; read الضح as in the MS.

thing was like unto it(self),/ like the time of 161:18
the day preceding for the afternoon prayer; then
the afternoon prayer when the shadow of anything
is twice itself,/ then the sunset prayer at its 19
(same) time as the other day; then the last, the
nightfall prayer when (i.e., up to)/ a third of 162:1
the night has passed; and the morning (prayer)
when it dawns." And he said: "The prayertime
falls in between".

ʿUmar b. al-Khaṭṭāb wrote to his governors 2
in the seventeenth year of the Hijra:/ "Pray the 3
noon (prayer) from (the time) when the shadow is a
cubit up to (the time) that the shadow of anyone of
you is equal to itself;/ and the afternoon (prayer) 4
when the sun is elevated pure white the amount
(of time in) which a rider goes two *farsakhs*/ or
three; and the sunset (prayer) when the sun sets; 5
and the nightfall (prayer) when the [sun][1] disap-
pears/ up to a third of the night; and the morn- 6
ing (prayer) when the stars are barely visible."

He wrote to Abū Mūsā al-Ashʿarī: "Pray the 7
noon (prayer) when/ the sun declines; and the af- 8
ternoon (prayer) when the sun is pure white before
it begins to become yellow; and the sunset/ when 9
the sun sets; and delay the nightfall (prayer) if it
(the twilight) is incomplete; and the morning
(prayer) when the stars are (yet) manifest,/ and 10
recite long *sūras* from the detailed (book, i.e.
the Qur'ān)".

It is reported of him that (he said) with re- 11
gard to the breaking of fast, "Do not break the fast
until you see the night/ on knolls". Namely until 12
the darkness of the night [flows over][2] the small
mountains, but he/ should have referred to the big 13
ones, since the small ones are subordinated to them,
for it is possible that/ the sun disappear from the 14
small ones and its light (still) shine on the tops
of the big (ones).

It is said that Jaʿfar b. Muḥammad al-Ṣādiq 15
was asked about the times of prayer./ So he said 16
"God set the times of prayer at marks occurring in
the heavens,/ and (at) the variation of conditions 17

[1] Text الشفق ; MS الشمس.
[2] Text تسبل ; MS تسيل.

which are in the heavens so that observation of 162:17
those things yields definite, known/ borderlines by 18
which they are distinguished from other things, and
it is a virtue to pray as near as possible to these
borderlines, and to observe their times,/ and to 19
look for their occurrence. So he set the time// f.227a
of the sunset (prayer) at the setting of the sun,
and the nightfall (prayer)/ at the disappearance 163:1
of the twilight, and the morning (prayer) at the
rise of the dawn, and noon at the declining of the
sun/ and its transferring (of itself) from the east- 2
ern side to the western side. If the shadow be-
comes longer, then it is/ the time of the afternoon 3
(prayer), for which there is no (fixed) sign in the
heavens similar to those four signs,/ so He made a 4
limit for it which has a wide range."

He said that (it extends) from the (time when 5
the) shadow of anything becomes equal to itself up
to (the time that the shadow becomes) [twice][1] it-
self. (But) on another occasion he advanced it/ and 6
combined the noon and afternoon (prayers), but on
another (occasion) he delayed it saying that it may
be prayed as long as/ the sun continues to be pure 7
and white.

It is reported of him also that he said: 8
"The precepts of the prayers are at/ odd hours: the 9
noon (prayer) at the beginning of the seventh, and
the afternoon (prayer) at the beginning of the ninth,
and the sunset (prayer) at the/ beginning of the 10
first (hour), and the nightfall (prayer) at the be-
ginning of the third (hour), and the morning (prayer)
at the beginning of the eleventh".

With regard to the inspired scripture (i.e. 11
the Qu'rān) God, be He exalted!, said,

> Pray at the two extremes of the day/ 12
> and the first part of the night,
> for verily good deeds compensate
> for the bad ones[2].

But the day(light) is a length of time,/ and the ex- 13
tremes of a length of time are two, (even) as two

[1] Text مثلها ; read مثليها as in the MS.
[2] Qur'ān xi,114. Text قم ; read اقم.

points are for a line, and the durations of time 163:13
are not/ long enough to carry the action and to ob- 14
serve the religious ordinance, and praying at a time
near the extreme is what/ is referred to. The day- 15
(light) cannot actually be divided except into two
halves at the culmination of/ the sun. Then for the 16
rest of its fractions there is no sign, and for them
recourse is had to imagination./ So the two parts 17
of the daylight are its two halves. Then the day-
(light), even though it is naturally begun by the
rising of/ the sun, is taken in canon law as the 18
beginning of the time of fasting. The dawn prayer/
falls at one of its two ends, and the noon and af- 19
ternoon (prayers) at the other end, while the sun-
set and the nightfall (prayers fall)/ in the night.

The Exalted One (God), said, 164:2

> So glorify to the praise of thy Lord
> before the rising of the sun and
> before/ its setting[1]. 3

These are two parts of the day, and what occurs be-
fore its (the sun's) rising is the sunset (prayer)
and the nightfall (prayer) and the dawn (prayer;/
and what is before its setting is the noon (prayer) 4
and the afternoon (prayer); and what is after that
is a repetition of the command, and details/ what was 5
stated briefly. It may be (taken) that what is be-
fore sunrise is the dawn (prayer), and what is be-
fore/ its setting is the afternoon (prayer) and what 6
are during the night are the sunset and nightfall
(prayers), while the ends of the day (are at)/ noon, 7
because it is the meeting (point) of the two proper
halves, which deserve the name end. And (He) said,

> So glorify to the praise/ of thy 8
> Lord when thou arisest, and at
> night glorify Him when the stars
> are disappearing[2].

So (is) the arising from the siesta,/ which implies 9
the noon and afternoon (prayers) together because of
the conjoining of their times, while the night

[1] Qur'ān xx, 130.
[2] Qur'ān lii,48.

includes the sunset/ and nightfall (prayers) 164:10
because conjoining them (is possible). Dawn
occurs when the stars are disappearing. (God)
the Exalted/ said, 11

> Pray at the declining of the sun,
> and at twilight, and the
> appearance of the dawn.[1]

The (period of time) beginning from the declining 12
of the sun until the twilight disappears includes/
all the prayers, except the dawn (prayer), and 13
hence He mentioned it separately.

The people took the "declining" (*dulūk*) as 14
being the sunset, taking (the word as being asso-
ciated with)/ the eye because of the disappearance 15
of the sun at that time. This explanation is not
recommended, because the sun,/ at the time of (its) 16
rising and setting is most apparent because of the
vanishing of its (strong) rays making one close/
the eye. So how can sight require this unless it 17
(the word *dulūk*) refers to the nightfall prayer! If/
this reasoning is true then noon will no longer be 18
(so called) because the strength of the sun's rays
and the eye's need of being/ rubbed when one arises 19
from midday sleep, so that vision is straightened
out.

So let us now mention what has been said 165:1
about the names of the prayers, because they are
adjunct to their times./ The noon (prayer) is 2
from noontime, which means the violence of the heat,
and the majority regard it as being taken from *al-
ẓahīr*, it being/ the (name of) the strong-backed 3
camel, and because the sun, with its rays spreading
out, is then most apparent./ The afternoon is the 4
night (*or* weakness of sight) and it can be used in
connection with more than one (thing), so one may
say the two afternoons (*ᶜaṣrain*). Similarly/ the 5
night and the day are the two afternoon also.
The Prophet, the prayers of God upon him [and
peace]![2], said to Faḍāla/ al-Zahrāni, "Observe the 6

[1] Qur'ān xvii, 78.
[2] وسلم not in MS.

two afternoon (prayers)". But this was not his 165:6
(Faḍāla's) dialect. So he asked him about them./
So he said, "The prayer before the rising of the 7
sun, and the prayer before// its setting", and f.227b
between the two of them/ in the statement, "Be- 8
cause they are truly, with respect to day and
night, linked together and similar (with respect
to)/ the two times, like calling the morning and 9
the night the two cold ones". Thus spoke Ḥumayd
b. Thawr,

> You cannot bear the shadow on a cold forenoon, 10
> Nor enjoy the shade on a cold[1] evening.

Verily this is impossible except in a bit- 11
terly cold winter.

It is said of the afternoon prayer, "Verily 12
(the word) *ʿaṣr* means killing for the extraction of
anything",/ as if it is killed and wrung out by be- 13
ing delayed.

Some people hold the opinion regarding noon 14
(*ẓuhr*) that it is (thus) called because of the
straddling of the sun on the back (*ẓahr*) of/ the 15
cupola (i.e. the celestial hemisphere), and that
its declining from it is like its going for pros-
tration to God, the Exalted, and hence it became/
a time for prayer in the afternoon inasmuch as 16
the sun was wrung (down) from the convexity of the
cupola,/ and its reaching in descent the place of 17
kneeling.

As for the night (prayer), it is after the 18
ʿashī, which is from noon until (the sun's) disap-
pearance,/ namely it (the prayer time) is from the 19
setting of the sun until the first part of the
night has elapsed, and its origin is the receiving
of/ darkness, because the light of the night fire 166:1
takes place during the darkness.

Verily the sunset prayer is called the first 2
night (prayer) because of its being in the first/
of the period of time (above-)mentioned, and the 3
nightfall prayer is called the other night prayer
because it is in the latter part of/ that period of 4
time, being delayed until the darkness is completed

[1] برد not in MS.

by the disappearance of the twilight, and "dark- 166:4
ness" (*iʿtām*) means/ the act of being delayed. 5

Al-Shāfiʿī and Mālik dislike calling this 6
prayer the darkness (prayer) because/ God the Exal- 7
ted called it the night prayer, even though the
name tends to be used in connection with both, the
first/ and the other one. But it is related thus, 8
"Let not the desert dwelling Arabs precede you in
prayer"./ By that is meant the night (prayer) when 9
they drive the camels home.

So, let us return to the times of prayer. 10
We say that the written prayers are/ actually di- 11
vided into two main categories: ***The daylight ones***,
which are speechless, in which one murmurs, except
for what is/ excluded by the guide (?*dalīl*) such as 12
on Fridays and on the two festivals (i.e. *al-Aḍḥa*
and *Ramaḍān*) at the appearance of Islām, and upon
the defeat/ of the infidels. Because the murmuring 13
is due to hiding of the Prophet, upon him peace!,
with/ the faithful in a house, and the cruel injury 14
done him by the infidels. ***The nocturnal ones***, in
which loud recitation is performed./ The dawn 15
prayer is mentioned separately so that none may be
misled by the common habit/ of considering it as 16
part of the night.

As for the nocturnal prayers, the first of 17
them is the sunset (prayer), and the beginning of
its time is the setting of/ the sun, its setting 18
being the disappearance of all of its disk under
the earth without (any) barrier between it/ and the 19
observer such as an object protruding from the face
of the earth, or standing between it and the sky.
As/ for its time, according to al-Shāfiʿī there 167:1
is no duration of time for carrying it along, de-
laying its (start) until the end of it, for its
time is/ one instant, its (duration) being the 2
amount of time one prays the sunset (prayer) after
the setting of the sun.

However, according to Abū Ḥanīfa and his as- 3
sociates, for its time the first of it is the set-
ting of the sun/ and the end of it is the end of 4
the twilight, except that the twilight, accord-
ing to Abū Hanīfa is the whiteness/ while accord- 5
ing to Abū Yūsuf, and Muḥammad b. al-Ḥasan,

and al-Shāfiʿī, it is the redness. But Aḥmad/ 167:5
ibn Ḥanbal took for security the end of the sun- 6
set (prayer) to be the redness in the open air/
and in the desert, and the whiteness in built-up 7
regions in the space between the buildings, be-
cause the redness is contiguous with the horizon,/
and it is obscured by objects. 8

But Mālik maintained that it is the arising 9
of the dawn, and it occurs between the disappear-
ance of the twilight/ and the rising of the dawn, 10
making in common the two prayers of sunset and
nightfall.

The second prayer of the nocturnal prayers 11
is the prayer of nightfall. The beginning of its
time/ according to all is the disappearance of the 12
twilight, (however) there is disagreement as to
what it is. The end of it is the rising of/ the
dawn. Its postponement to one third of the night 13
or half of it is to be considered from the point
of view of convenience, not/ of time. 14

The third prayer of the night prayers is 15
the prayer of dawn, and the beginning of/ its time 16
is the rising of the second dawn// after the f.228a
false dawn (*ṣubḥ*) and they do not disagree as to
the dawn being/ the whiteness spread out along the 17
horizon in width after the rectangle erected per-
pendicular to it/ resembling a wolf's tail. Some 18
say it is the green[1] which precedes sunrise. They/
disagree as to twilight, even though these dif- 19
ferences arise because of the position with respect/
to the sun on its two sides and with respect to 168:1
the horizon in its regions.

There are differences as to which is pre- 2
ferable. Al-Shāfiʿī maintains concerning it that
it should be small and/ barely less[2] than a cubit. 3
However, this cubit should be additional, over and
above half the noon (shadow)/ itself, otherwise the 4
(above) rule will collapse (if applied) at different
times and places.

---

[1] Text الخضرة ; MS الحمرة.
[2] Text نزر القصر ; MS نزرايقصر . The change of subject at 168:2 indicates a hiatus in the MS.

The first point of view is more worthwhile 168:5
in so far as the above statement is concerned, and
more pertinent, since it does/ not mention for the 6
afternoon the noon shadow together with its equal
and twice itself. It is clear that that time (of
the year)/ requires the diminishing of the shadow 7
and its vanishing. So that when it (the shadow)
appears to the amount of the rope of a trap, this/
is an indication of noon. Had it had any magnitude 8
it would have been mentioned with its equal and twice
itself./ In most localities it exceeds at noon the 9
equal (of the gnomon length) on most of the days/ of
the year. But if it is not mentioned when it is 10
discussed, noon will be the beginning of the time
of the afternoon (prayer). It is even possible/
for noon not to be the time (for the beginning of
the afternoon prayer) when its endpoint fails to 11
exist. (This endpoint) is, according to the well-
known traditions from/ Abū Ḥanīfa, the time when 12
the shadow of anything is twice itself, after the
noon shadow. And it is related of him/ also as 13
having replaced the twice by one, which is the say-
ing of Abū Yūsuf and Muḥammad and al-Shāfiʿī/ also. 14

The second prayer is the afternoon prayer. 15
The beginning of its time is the end of the time of
noon time,/ and hence it is related to the (time 16
when) the shadow of anything is twice itself after
the noon shadow, according to/ Abū Ḥanīfa in the 17
best-known traditions, or the equal of it in the
other tradition and according to/ Abū Yūsuf and 18
Muḥammad and al-Shāfiʿī.

It is related of Abū Ḥanīfa in some of the 19
traditions that if the shadow becomes/ equal to 169:1
the thing (casting it) after the noon shadow,
then the time of noon (prayer) has run out, but
the/ time of the afternoon prayer does not commence 2
until the shadow becomes twice the thing (casting
it), after the noon shadow. But/ this tradition is 3
not of the well-known ones, and the time of after-
noon (prayer) according to al-Shāfiʿī lasts until
the shadow is/ twice (the object). If one delays 4
(praying) until after (this limit) this is a matter
of choice.

It is reported from ʿAṭā' and Ṭā'ūs, and 169:5
Mālik regarding the end of the time of afternoon (prayer)/ the direct explanation of the Revealed 6
(Qur'ān) in the saying of (God) the Exalted, "Pray at the declining of the sun and at/ twilight", as 7
well as (the fact that the endpoint) can be delayed as long as the darkness of the night. Thus the act of/ increasing the shadow by one (gnomon) length has 8
to be done with respect to noon and (likewise) afternoon, the time/ preceding that being considered as 9
the noon (prayer).

As for the times in which prayer is forbidden, 10
they are when the/ sun is on the horizon and in the 11
meridian, as we mentioned previously.

The disapproved (but not absolutely forbidden) 12
times (of prayer) are at the reddening of the sun and its becoming yellow/ after its rising until its col- 13
ors clear up and its rays blaze vehemently. Also before its setting/ such that the yellowness of its 14
body is considered, but not (that of) its rays which fall on/ a wall, or on mountaintops. The supererog- 15
atory (prayer) is also forbidden for those who pray the morning prayer/, and for those who have prayed 16
the afternoon (prayer) until its setting. But the/ required prayer is not forbidden (then) if one has 17
not (yet) prayed.

Verily it is mentioned in the traditions, "Do 18
not delay prayer until the death rattle (*sharaq al-mawtā*)"./ Abū ʿUbayd said, "It is the time in which 19
the sun's (rays) are high on walls and fall upon/ the graves when it (the sun) is weak, and when it 170:1
is on the western side with a clear horizon without any/ obstacle in a locality, and the graves in it 2
are on the side, with (all the above-)mentioned occurring"./ He restricted the matter to the after- 3
noon prayer, whereas this (term *sharaq*) has nothing to do with sunrise (*sharq*)./ Thus it should (not) 4
indicate the morning (*al-ghadāt*) prayer (only). Neither is it related to the afternoon (prayer alone) without the/ other prayers. Rather it is from "the 5
choking obstruction" (*al-sharaq b'il-ḥarīḍ*). Probably it is the dropping (of medicine) in the throat at the/ last of the pangs of death, and to it (*al-* 6

ḥarīḍ) is given the name// of death. This is f.228b
indeed a command not to delay (any)/ prayer of 170:7
the (prescribed) prayers in general up to the last of their periods which resemble the last (living) time of the one dying,/ at which is the 8
decline of (life's) duties. It is incumbent upon one to pray during the most extended of their periods,/ which resemble life, and before the sun 9
expires with its redness and weakness, which is called at/ that time the death rattle. This is 10
(also) emphasized by his (the Prophet's) saying, upon him peace!, "Pray/ as long as (the sun) con- 11
tinues to be pure white and alive". So if the whiteness is its life, the/ redness is its death, or its 12
death pangs, if the setting is truly resembling its death. And from this is (the expression)/ "red death". 13

The poet said, 14

> When the western prospect is rich with blood 15
> Due to the red at the sun's being assassinated by the horizon.

So these are the opinions of the past gen- 16
erations of the *imāms* of Islam concerning the times of prayer./ Of the Shiʿites there are those who 17
count the times of the twilight (prayer) as among the day (prayers), while others consider/ the dawn 18
(prayer) as among them also, taking the time of the sunset prayer as the disappearance of the twilight, and the time of/ the prayer of dawn as its 19
rising, while the time of the nightfall prayer (is taken) as midnight. And it is reported from/ their imāms, their saying; "Delay the sunset (prayer) 171:1
until the sun sets, since it (the sun) hides behind/ the mountains". Among them are those who fix 2
the beginning of the night by the (first) visibility of the star(s). So they make it the time of the sunset/ prayer. They terminate the fast as though they 3
consider the appearance of the stars as corresponding to their disappearance (for beginning and ending the night).

The Zaydites say, "Pray it with the departure 4

of the redness, and if thou seest a star,/ then 171:5
pray and break the fast, for God says, 'When the
night was all around him he saw a star'".[1] He
said,/ "The night being all around does not imply 6
the (actual) existence of night with which the
breaking of the fast is associated".

The extremists of them said, "When the two 7
bright stars of Ursa Minor rise, then pray and break
the fast", but anyone who knows/ that the fixed 8
stars differ in magnitude, realizes that their
(first) visibilities differ in time,/ moreover, if 9
Jupiter is near the condition called the end of the
night, it is visible at/ sunset because it is (then) 10
in the blackness which begins from the east at that
time, it being the beginning of/ the darkness of the 11
night. Whereas if Venus is at the farthest of its
distances from the sun/ along the (direction of the) 12
succession of the (zodiacal) signs, it is seen be-
fore sunset, so that to relate the (beginning of
the) night to the visibility of/ the stars is an un- 13
reliable opinion, not to be taken into account.

As for the followers of Abū ʿAbdallāh b.Karrām, 14
I know some of them who want to/ take for the time 15
of the afternoon (prayer), a middle position among
(all) those opinions, preferring the (golden) mean./
But these (people) do not agree on this middle posi- 16
tion. Shall they do it for the shadow, taking it
as one/ and a half (gnomon lengths) in excess of the 17
noon shadow? Or shall they take it as the time mid-
way between/ the two opinions? And each of these 18
two middle positions for the shadow and the time
differs/ from the other by a (certain length of) 19
time, and if they ever coincide it is by chance.

As for the books, which contain their 172:1
choices in canon law, they do not agree/ except 2
with the opinion of Abū Ḥanīfa in increasing by
twice (the gnomon length).

Among the appellations of the prayers are the 3
"first" and the "middle" prayers. As for the first
there is no disagreement/ as being the noon prayer, 4
because it is the first of the day prayers, and the
first to be assigned/ and prayed and, it is said, it 5

[1] Qur'ān vi, 76.

was made manifest. It is also the first to be 172:5
encouraged by His saying, be He exalted!,/ "Pray 6
at the declining of the sun". And from this (prayer)
begins the order (of prayers) as was mentioned
in the/ information of the teaching of Gabriel con- 7
cerning the times of prayer. It is even reported/
that the governor of the Ṭā'if asked a desert- 8
dwelling Arab for (the sake of) emphasizing (the
times of prayer) about the number of times/ he prays 9
in a day and a night. So he said:

> Verily the prayers are four and[1] four. 10
> Then three followed by four;
> Then the dawn prayer, which should not be 11
> missed.

So he began the order with the noon (prayer), 12
which is well-known as the first.

As for the middle (one), they disagree con- 13
cerning it, and they explain it in so many ways//f.229a
some of them attain the limit of the ridiculous, to
the extent that it is said that the meaning of the
middle is the most important for its being virtuous/
and most rewarding. And each one considers it dif- 15
ferently, that it includes all the/ (above-)men- 16
tioned prayers except the night one which was com-
pletely ignored.

It is reported of ʿAlī ibn abī Ṭālib, Ibn 17
ʿAbbās, Qatāda and Mujāhid/ that the middle one is 18
the morning prayer, and they hold the opinion about
it that the recitation of the dawn (prayer) is wit-
nessed by/ the day angels as well as by the night 19
angels. Thus it is intermediary/ among them. 173:1
It is also odd and cannot be paired by any other
one, such as pairing the noon and afternoon prayers/
at ʿArafāt, and the pairing of the sunset and the 2
nightfall (prayers) at Muzdalifa, and the act of
being odd is preferred to the act of being paired./
Al-Shāfiʿī inferred this by mentioning the *qunūt* 3
with it. But the *qunūt* should be used/ only at the 4
morning prayer. The explanation here is taken from
the structure (of the word) and not the meaning (which)
allows the *qunūt* to be used with/ any prayer, 5

[1] Text فاربع ; MS واربع .

according to His saying, be He exalted!, Who is 173:5
he who, during the night, uses the *qunūt* and pros-
trates himself?"/ Jābir b. ʿAbdallāh maintains that 6
it (the middle prayer) is between the darkness and
the light, since thus it is midway between/ the two 7
threads. It is reported of ʿAlī b. abī Ṭālib in
this connection, "Verily it is the middle between/
the two prayers of the day and the two prayers of 8
the night". But this implies that he did not con-
sider it as belonging either to the night/ or to 9
the day, as is the case in their sect (the *ʿAlawī'īn*)
concerning the time of dawn and twilight,/ which are 10
made midway betwen day and night but not belonging
to/ (either) one. ʿAbdallāh b. ʿUmar maintains, 11
with regard to the middle (one) that it is the noon
prayer/ because it is (at) midday. 12

Others assert, among whom is Qubayṣa b. 13
Dhuwayb that it is the sunset prayer/ because it is 14
the mean (one) between the longer, which is four
genuflections, and the shorter, which is/ two genu- 15
flections, and that it deserves the virtue of being
odd in number, as well as the meeting of/ the day 16
and the night angels at it, and that the inclining
of the sun is one end of its time. But they left
the nightfall prayer/ out of consideration, it being 17
the middle (one) of the nocturnal prayer(s).

However, the sound opinion in the matter 18
is that it is the afternoon prayer, which is well
known as the middle,/ so that this has become an 19
epithet and a nickname for it generally, and be-
cause it is the middle (one) between the two/ day-
time prayers and the two night prayers, accord- 174:1
ing to those who commence the day with dawn.

However, as to the reason for mentioning 2
it in particular, it is in order to repeat the com-
mand for observing it, because/ its time, according 3
to the habits of the people is sacrificed for work
and the ending of/ the duties of the day and the 4
commencement of the duties of the night. At such
a time it is quite possible/ to forget about it. 5

It is related of the Prophet, the prayer 6
of God upon him and peace![1] that he prayed the

[1] وسلم not in the MS.

afternoon prayer with us, and then he said:/ 174:7
"Verily, this prayer was presented to those who
preceded you, but they forgot about it. He among
you who observes/ it, his reward is doubled".[1] 8
Moreover since the times of the rest of the prayers/
have (fixed) signs, which are quite manifest, such 9
as morning, midday/ sunset, and the disappearance of 10
twilight, but this is not the case with the middle
prayer./ Because its sign is in the hearts (*lit.* 11
chests) in contrast to them (the preceding), and
(it involves) noon as well as its numbers. The
general command/ of observing the prayers is to pray 12
when their times are due[2], and their signs and/
marks are apparent. The afternoon prayer is in- 13
cluded (there too) among them, but the special com-
mand in its case involves finding its/ time and 14
observing its signs.

Although we dispense with (detailing) the 15
times (of prayer) of the other faiths, (bear in
mind) that mentioning them (would be)/ a type of 16
knowledge of which it is harmless to be aware.
The prayers of the Jews, although/ their books of 17
Moses, upon him peace!, the Pentateuch, are devoid
of commands concerning prayer,/ are not nocturnal. 18
They are three. The first is at sunset, the second
at/ dawn (*saḥar*), and the third in the morning when 19
a white thread can be distinguished from a black
(one).// Each/ one of them (involves) eighteen f.229b
genuflections (*rakʿa*). The prayers of the Chris- 175:1
tians are seven. They are: midnight,/ morning, 2
the forenoon, noon, the afternoon, sunset, and
nightfall.

The prayers of the [Manicheans][3] for the ini- 3
tiates (*or* elect, *ṣiddīqīn*) are seven: The first
of them is the prayer of the vertical, at/ noon, 4
(having) thirty-seven genuflections, but on Monday
they are decreased by/ two genuflections. Then (2) 5
the afternoon (prayer) with twenty-one genuflec-
tions; then (3) the nightfall (with) twenty-five/

[1] Text يؤتى ; MS التسوف.
[2] Text يفعلهااذاجاز; MS يفعلها اذاجار.
[3] Text الثانية ; MS المنانية.

genuflections, then half an hour (after) (4) 175:6
night(fall), an equal (number) to it; then (5)
midnight, thirty genuflections;/ then (6) dawn, 7
fifty genuflections; then the preaching at the end
of (7) the night and the beginning of the day,
twenty-/six genuflections. Their auditors (*or* 8
laymen, *sammā'ūn*), who deal with worldly (affairs),
pray four/ prayers, they being noon, nightfall, 9
dawn, and sunrise.

The prayers of the Magians are three(fold), 10
as we said (before) depending upon the sun('s po-
sition)./ They pray (also) to the moon once each 11
month, and in the presence of fire, to the fire.

Let us mention now what is needful for the 12
one who determines the times of prayer to say./
The situation being along the lines we described, 13
the signs of the prayers are (to be determined
from)/ the effects of their opposites at their 14
times. I mean that the reference for the two day-
time prayers is the shadow, and the shadow/ be- 15
longs to night, even though the sun is the indica-
tor for it. The references for the nocturnal prayers
are/ dawn and twilight, which appertain to the day 16
because of the light. And if a/ just man observes 17
the noon shadow he realizes that a man charged with
it must/ observe it assiduously each day through 18
the year until he comes/ out with the shadow of the 19
afternoon (prayer) for that (place), chosing it
from (among) the sayings of the *imāms*. (It is admit-
ted) that the/ matter of the (time of) exist- 176:1
ence of the shortest shadow for (that) day, is
not easy to observe in the absence of a scientific
rule/ for doing it. So, if one establishes the 2
magnitudes of the noon shadows for the days of the
year, they will be different (for the following
year)/ when continued if one follows the lunar year. 3
So it is impossible for one to record it except with/
the solar year. So if one wants to standardize 4
it it is necessary to use the Byzantine months/
and the knowledge of leap years among them. If one 5
does not admit imitation the affair is attended with
the difficulties of/ parts of days divided up for 6
their months, and that involves the equalizing of

the sun's travel/ in the heaven of the apogee, 176:7
which is a variable in our (system), as well as a
method of extracting that/ and observing it with 8
the armillary sphere, and (other) instruments.

It is also possible, but I hope it will not 9
occur, that the muezzin/ in charge may miss the noon 10
shadow on some day or on some successive days, for
reasons within his control or/ beyond it, entailed 11
by events on high (?). Indeed I was observing/ at 12
Ghazna the noon altitudes for an urgent inquirer
who was incompetent to perform it because of its/
difficulty. Then it happened that for successive 13
days of the year, in number near to twenty,/ the 14
sky was very clear until (just) before noon, but
as soon as/ the desired time arrived some scattered 15
clouds appeared and they joined and came together
making me miss[1]/ my objective, and it rained on 16
most of them, then behold, an hour after noon had
passed it became/ clear and the atmosphere pure. 17

If it happens for him as I stated he cannot 18
determine the shadow for the afternoon prayer by/
missing the noon shadow. If he is required to per- 19
form his duty he can either give up, or imitate/
the practitioners of the craft, any other alterna- 177:1
tive being ignorance and haughtiness/ and confu- 2
sion.

Then he should say that midday, which fixes 3
the time of the noon prayer/ is not to be determined 4
except by one of the (following) four methods: ei-
ther (1) it is the midpoint of the time between sun-
rise/ and sunset, or (2) it is the time when the 5
azimuth is (half way) between the rising point and
the setting point, or else (3) the (time of) the
sun's ending/ its upward progress on that day, or 6
else (4) the (time of) its shortest shadow. How-
ever the observation of the shadow/ and the alti- 7
tude with their (respective) instruments is a tech-
nical matter, the performance of which will not be
withheld from anyone who knows/ something// of f.230a
the (subject) matter of the two, or from anyone 8
who has read this book of ours.

[1]Text نقوتنی ; MS تفیتنی.

As for the determination of the azimuth 177:9
between the rising point and the setting point, I
mean the meridian line,/ it is the most useful of 10
these methods. As to its extraction by technical
methods,/ enough has been said. On its practical 11
side and its justification by proof there is need/
for a goodly section of the two arts of astronomy 12
and geometry, but it requires in addition/ conic 13
sections, which some call, because of their dif-
ficulty, spiritual geometry.

However, as for that which is connected with 14
the time (of prayer), it is well-known that guessing
about it, as is done by/ most of the callers to 15
prayer, is not trustworthy for it, and guesswork
does not go back to a law which will enable/ its 16
user to rely upon it when someone disagrees with him
about it by taking it as a witness and a proof. More-
over people/ differ from one another in degree as 17
to guesswork and intuition due to differences in
their temperaments. Training/ by using it frequent- 18
ly and persistently would have a better effect if
it were not for human cunning/ which spoils the idea. 19
This is that the human, when he is charged with an
affair, whether practical/ or theoretical, will 178:1
not be devoid of (some) thoughts, and the remem-
brance of (certain) situations which endanger/ his
heart for a time. It passes as the water of a riv- 2
er, through his consciousness and heart[1], it be-
ing a category, an example of which is/ dreams. 3
Discussion regarding it can be lengthy. (Indeed)
it is not possible to free the heart from it and to
compel/ the imagining force to forsake it, except 4
for a moment, after which it comes back. I will
satisfy myself with one/ incident, the stammering 5
of the majority of those who maintain the beliefs
of al-Shāfiʿī in the opening of/ the prayer, and 6
their strange hesitation in purifying their inten-
tions which are thus made impossible for them and/
useless for announcing the prayer-time. So if 7
guesswork is stricken with this disease, who can/
rely on intuition[2], and the guesser, believing 8

[1] Text على بالة ولبة ; read على باله ولبه .
[2] Text التفرس ; MS المتفرس .

his ability (to give) the correct times/ by 178:9
guesswork (taking into consideration the possibil-
ity) of his performing equally reliable operations,
or of repeating some statements of a certain sys-
tem,/ or of coming out with a number which is close 10
to one of the calculated times. More reliable than/
this guesswork is time measurement with an instru- 11
ment made for (measuring) a part of it (i.e. the
time), be it an hour,/ or portions of it, or mul- 12
tiples of it. Thus one determines from it the
length of half the arc of that day, whether/ it is 13
the instrument made so that water enters it or so
that water leaves it, or sand,/ or something else 14
like that. But this operation necessitates the
predetermination of the arc of/ daylight by compu- 15
tation. That is because its determination by an
instrument is not possible except after the deter-
mination of the whole (arc) of it./ And the obser- 16
vation of the whole arc of daylight cannot be ob-
tained except after the end of the day, all of it,
and that is not/ useful for the (determination of) 17
noontime, since its time will have passed.

But what is called for in this topic, after 18
the knowledge of the conditions of heaviness/ and
lightness and centers of gravity, which is based 19
on the science of the shape of the universe, is the
equation of daylight/ for each part (i.e. degree) 179:1
of the parts of the ecliptic at the locality as-
sumed. But the equation of/ daylight requires, for 2
the locality, its latitude, and for the ecliptic,/
the position of the sun and its declination. As 3
for the latitude of the locality, it results as the
mean/ between the altitudes on two days of one of 4
the never-setting stars, or the complements of/ the
means between the (maximum) altitudes (of the sun) 5
at the solstices, or the declination of the sun or
one of the stars./ But the declination of the sun 6
requires the observation of the inclination of the
ecliptic, then it is cut up into parts,/ and the 7
declination of a star calls for the observation of
its position in longitude and latitude,/ and both of 8
them need computation of sines and chords and ascen-
sions. For the solar/ position we need the knowledge 9

of calendars of the (various) peoples, their years 179:9
and their months,/ up to the observations of the an- 10
cients and the moderns, finding from them the solar
positions by/ the mean motion and the variable (mo- 11
tion) and the amount of the difference between the
two. And by this is determined the/ arc of daylight 12
of any day desired.

However, as for the extreme altitude of the 13
sun in largeness and of the shadow in// small- f.230b
ness/ and their observation, it is evident that the 14
difference of altitude around noon will be/ in parts 15
of parts due to the fact that large instruments can
hardly register it (accurately), much less/ the small
(ones). Hence it is supposed that the sun at that 16
time is stopped because the altitude of the sun/ and 17
its azimuth is constant, inasmuch as can be perceived,
at one amount. So the usefulness of the predetermi-
nation of/ this altitude in order to compare it 18
with the existing (one) is not so great for preci-
sion, since one, for/ its determination, (falls 19
back) on the solar position, and the inclination of
the ecliptic, and the local latitude, and thus/ at
noon one needs what was needed for the noon alti- 180:1
tude plus the/ determination of it (noon) from it 2
(the shadow).

So, if the muezzin is interested in deep in- 3
vestigation, and he abstains from (blind) imitation,
and (if)/ his temperament is akin to the science of 4
Ptolemy, and Archimedes, and Apollonius, and he
never puffs himself up above/ these names, and he 5
seeks schooling and education until he reaches this
position,/ then verily he must take up the whole of 6
the Book of the Elements (of Euclid) and the middle
works between it and the Almagest,/ and he must give 7
(himself over) to eight treatises of it. Thus he
came as empty as the devil, but he goes away as vic-
torious/ as (the prophet) Enoch (*Idrīs*). If it hap- 8
pens that he becomes fed up from the very first with
studying what we have mentioned,/ then let him take 9
the shortest distance away from the work, let him
shorten the length of hope by giving the bow over to
one who can draw it and surrendering/ the matter to 10
the experts who do not loathe steady striving for the
reform of these/ elements and their improvement, and 11
the production of their results to those who seek them.

THE TWENTY-SIXTH (CHAPTER) 180:12

ON THE ESTABLISHMENT OF THE LINES FOR THE TIMES OF PRAYER AND THE HOURS ON INSTRUMENTS 13

The dependence of the matter of the two 14
times, of the noon and afternoon (prayers), upon shad-
ows, has been (by now) clarified. As for noon, it
is because/ its time follows immediately the decline 15
of the sun from the meridian, since the shadow of/
the gnomon (*shakhṣ*) on the horizon plane, if it is 16
found to be equal to the shadow of the (solar) alti-
tude for noon of/ that day at that locality, then 17
it is midday and noon, at which (time)/ the prayer 18
is forbidden. Then follows immediately the begin-
ning of the time of the noon prayer when the shad-
ow increases from that/ amount by something, then 19
it is the time of the noon prayer according to what
we mentioned, which is easy/ to visualize in 181:1
imagination (but) hard to use in practice.

Hence, it is directed, for the determination 2
of noon, to erect a stick and observe its shadow/
after the time of erecting (it in comparison with 3
its length at the)[1] time, and if it is less than
the first amount/ the time will be before noon, but 4
if more it will be after it. This, as to its cor-
rectness, is close to being/ suspect for one who is 5
not practiced in this art. For firstly it may be
found at/ two times, the both being equal if it is 6
around noon at two equal distances,/ and so it will 7
not lead to what is needed by this (method).

Secondly, since the second time will perhaps 8
be nearer to noon/ than the first, in a direction 9
other than it is from it, I mean that the second is
after noon and its shadow/nevertheless is less than 10
the shadow of the first, which is before it, and one
might think that in spite of the passage of the me-
ridian/ it is yet to come. 11

Thirdly, if the procedure is not known of 12

[1] Text النصب من ; MS النصب بيسير من .

using the circumference of a circle described/ 181:12
about the base of the gnomon, then one might think 13
that a difference in the azimuth should indicate
something about the shadow, giving it an/ increase 14
or decrease which is not really present.

And fourthly, even if there is no error of 15
that (sort), one cannot tell by means of it, except
that/ the first time is before noon, whereas the 16
second can be (any) one of the three situations of/
being before noon, after it, or [just][1] at it, i.e. 17
at noon.

But fifthly, the difference of the shadow at 18
noon, especially in/ the summer, in localities of 19
low latitude, becomes imperceptible at intervals of
small/ amount, because the motion of the shadow 182:1
is that of the head of the shadow at the vertex of
the hyperbola// and/ its neighborhood. And f.231a
these are the reasons which, in the matter of (the 2
determination of) noon recommend/ the (use of) the 3
Indian circle, which was previously mentioned, and
the line in it extended between north and south,/
which, when reached by the shadow of the gnomon erec- 4
ted at it(s center), then it is noon, and if it/
exceeds it, even if it is by the smallest thing, the 5
time of the noon prayer has begun.

As for the time of the afternoon prayer, we 6
extract the noon shadow for that/ day, as before in 7
the chapter (devoted) to it, and we put it in two
places. And we add to one of them the like of/ the 8
parts of the gnomon, and it will be the shadow of
the afternoon-time according to Abū Yūsuf, and
Muḥammad,/ and al-Shāfiʿī, and we add to the second, 9
twice the parts of the gnomon, and it will be the
shadow of the afternoon-time/ according to Abū 10
Ḥanīfa. And if we want the altitudes at these two
times, their shadows having been obtained,/ we ex- 11
tract the altitude from the shadow in the manner
preceding in the chapter (devoted) to it. And this
(following) is what is in/ the *zīj* of Ḥabash. 12

He says in it, "We take the noon shadow and 13
add to it sixty parts/ after we transform the shad- 14
ow from the twelve (unit) type to the sixty (unit)

[1] Text حاقة ; read حافة ?

type. Then we find its corresponding arc/ in 182:15
the shadow table, and what comes out for the alti-
tude we subtract from ninety, and there remains the
altitude/ of the beginning of the afternoon. And 16
for the end of it we add to the noon shadow, after
the transformation, a hundred/ and twenty, and of 17
what results we find its corresponding arc in the
table of the shadow, and we subtract its arc from
ninety,/ and what remains is the altitude of the end 18
of the afternoon". But this is obvious if it is
known that the gnomon which we use for it is/ of the
sixty-part type, and that the table in which we 19
find the arcs is set up for/ the reversed shad- 183:1
ow, (i.e. the tangent function), and hence we
find the shadow corresponding to the complement
of its altitude.

If anyone wants to make the lines of these 2
times, it is necessary for him, for facility, to/
predetermine the shadows and altitudes and azi- 3
muths for each one/ of them, degree by degree of the 4
ascending half of the ecliptic, I mean (the half)/
which is from the first (point) of Capricorn to the 5
last of Gemini, so that it will be ready at hand for
the time of/ operation. But in astronomical instru- 6
ments like the astrolabe, which is well-known, one
does not find/ all that is needed for the use of 7
the people.

So, let us begin with its (the astrolabe's) 8
interior. We say that it is possible to make on the
faces of its plates,/ between the eastern horizon 9
and the meridian (*khaṭṭ watad al-arḍ*), the line of
the beginning of the afternoon and its end/ by
placing each of the degrees of the ascending half 10
of the ecliptic/ from the rete at the altitude of 11
the beginning of afternoon as extracted for it from/
the westerly almucantars, and marking the position 12
of the opposite point of that degree on the face of
the plate. We also/ place (them) on it at the al- 13
titude of the end of the afternoon and mark the po-
sition of the opposite point. If/ that is done for 14
all the degrees of the half certain marks will ap-
pear in succession for these two lines between/ the

tropics of Capricorn and Cancer. Then the crafts- 183:15
man carefully joins them by arcs, the entirety of
which/ may be imagined as a single curved unbroken 16
line.

If he desires to distinguish the two (curves) 17
from the lines of the hours, he should put successive
points on them (i.e. make them dotted),/ or write 18
their titles on them. Then he is free to perform the
operation which we described, degree by/ degree, or 19
sign by sign, or by the divisions of the ecliptic
(*manṭaqa*) on that (particular) astrolabe.

If he wants to have two (circular) arcs, 184:1
he may shorten the work by performing the operation
for the equinoctial circle/ and (those of) the two 2
solstices only, as is done with the lines of the
unequal hours in the division of the (part) under/ the 3
horizon of the three circles (i.e. the tropics and the
equator) by twelve equal divisions, and the passing
of an arc through/ each triple of them corresponding 4
to the operation of circumscribing a circle about any
triangle./ If this were done with all of the circles, 5
then the corresponding points on them would not be
concyclic/, hence the matter of the unequal hours 6
(determined) by their lines on/ the astrolabe is 7
taken approximately. However, the equal (hours) are
drawn// by (using) the distance of the (compass) f.231b
opening/ of the horizon, at the end of each division 8
of the twenty-four divisons of the circle/ described 9
about the center of the plate with a distance (equal
to) that of the center of the horizon. And this is
completely correct.

By the two lines which we made for the two 10
times of afternoon (prayer) the time passed until
them (the two times)/ from the beginning of the day 11
or from noon is determined if the opposite of the
degree of the sun is put on them and at/ the position 12
of the pointer on the ring a mark is made, then the
rete is rotated backwards to/ the left until the 13
degree of the sun arrives at the line of midheaven
or the eastern horizon./ And so what the pointer 14
moves on the ring from the mark will be the duration
of the (time) passed.

Also the remaining (time) until the end 184:15
of the day is determined by rotating the rete/ to
the right evenly until the degree of the sun arrives 16
at the western horizon. When/ the altitude of the 17
sun is measured at a certain time[1] and its degree
placed on its almucantar, then/ by the position of 18
the opposite of the sun's degree from these two
lines is determined whether their times are due,/
or have passed, or are to come. Analogously to 19
this the line of the rising of/ dawn is construc- 185:1
ted on the plate [by putting][2] the opposite of the
sun's degree on the eighteenth almucantar always/
on the western side; and the line of disappearance 2
of the twilight by putting it on that almucantar/
on the eastern side. 3

Concerning what is said regarding ʿUmar b. 4
ʿAbd al-ʿAzīz, that he was making the call for the
noon (prayer) at the/ seventh hour, but he (some- 5
times) prayed this prayer at the eighth hour, and the
afternoon (prayer) at the tenth hour, (one should
remember that)/ these hours were unequal (ones) un- 6
doubtedly. Some of them (the people) shifted from
shadows/ over to their (the hour) lines taking the 7
line of the tenth hour among the/ unequal hour lines 8
for the end of the time of the afternoon (prayer),
and the line of the ninth for the beginning of its
time, just as the line of/ the third of them is for 9
the time of the morning prayer. But this is con-
trary to the religious law, and it should/ not be 10
followed.

Some of them take for the time of the noon 11
call to prayer (as being) when the increase of the
shadow is/ one digit, and the time of arising (from 12
prayer) is when the increase is three digits, and/
the afternoon when the increase is thirteen digits. 13
And if we agree with them as to the time of the noon
call,/ then the increment equal to it will not be 14
over its shadow but over the noon shadow itself,/
but the matter is not up to them. It is the result 15
of ignorance about digits, which are/ halves of 16
sixths of a gnomon, whether it be a span or if the

---

[1] Text لوقت ماو ; MS لوقت و .
[2] Text يوضح ; read بوضع .

perpendicular is Mount/ Damāvand, for example. 185:17
And the (above-)mentioned digits for the noon call
to prayer should mean *a fortiori*/ the fingers of 18
the hand, since by the smallest of them the noontime
begins. The/ (above-)mentioned digits for the af- 19
ternoon time are mixed up with this unit and with/
the twelve parts of the gnomon. 186:1

Now let us consider the back of the astrolabe 2
in order to lay out these lines upon it,/ and let us 3
first put an edge on its alidade by cutting it length-
wise so that its edge will pass through the center./
But leave (some) of it around it (the center) in or- 4
der to cover the base of the pole, and we discard
the remainder of/ one of its two halves and we em- 5
place the two sights on the remaining half.

And let circle *ABGD* (Figure 47) on the back 6
of the astrolabe be/ that which is under the parts 7
of the altitude, and its quadrant is *AB*, and *A* on
it is near the socket (*or* throne, *kursī*),/ and arc 8
ZH is what the edged alidade covers on the back of
the astrolabe./ We divide line *DZ* into six equal 9
parts, and we write from the two sides/ the names 10
of the signs divided off in the ascending half and the

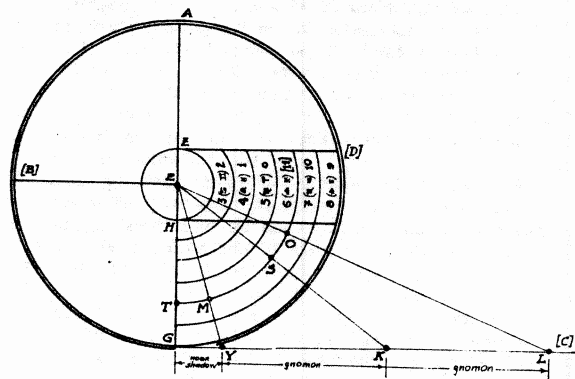



Figure 47

descending half in the/ form which we wrote 186:11
(before), and we divide each sign in a suitable
number of parts indicating/ the degrees of the 12
signs, depending on the size of the piece, and we
do not squeeze them together. We describe about
center/ *E* and at the distance of each division of 13
the quadrant *D*[*G*]*E*[1] a visible arc, one which is
not permanent,/ so that we can deepen of it what- 14
ever we need. This is the circle of the degrees.
Then we divide *EG*/ into twelve equal parts; they 15
are the digits of the gnomon. And we extend *GC*/
tangent to the circle and unlimited in the direc- 16
tion of *C*.// And we divide it in divisions equal f. 232a
in/ amount to the divisons of *EG*. And *GC* will be 17
for the shadow. Then we divide each/ circle. 18

Thus, for example, let the circle of the 19
first of Pisces and its end be line *EG*/ at *T*. 187:1
And we lay off *GY* equal to the noon shadow, and
we join *EMY*,/ cutting the circle at *M*. It is 2
the mark of the time of noon. We lay off *GK*/
equal to the shadow of the beginning of afternoon, 3
and *GL* equal to the shadow of the end of afternoon,
and we join/ [*E*]*SK* (and) *EOL*, and there will be two 4
points, *S* (and) *O* for the two times of the after-
noon (prayer).

We do thus for each circle, fixing for each 5
one three points/ for these times if we join cor- 6
responding (points) which are for a single line by/
convex lines (arranged) in order. Thus a line will 7
result on which we write its title. Upon finishing/
the three we put a permanent line for each circle 8
of that part which falls between the signs and the
line of the/ time of noon. We efface the extra part 9
of it in the direction of line *EG*, I mean arc *MT*./
And thereby is the operation completed. 10

If the edge of the alidade is placed along 11
the intersection of the sun's (daily) path at such
and such a time/ and the line whose time is required 12

[1] Text دحه; read دجه. On Figure 47 *B* is missing in the edition, but present in the MS. For *C* the editon has a second *S*, and *D* is missing. The 11 is restored from the MS; the edition has a second 10.

then the [pointer][1] of the alidade falls along 187:12
the altitude of that time./ Then the altitude 188:1
is measured at that time, and if it is more than
the altitude of the time, then it is not time for
it/ yet, but if it is less its time has already 2
passed, unless it is at the meridian, since (the
fact of being) less/ than it does not necessarily 3
indicate (the fact that) it has passed or that it
is to come.

Likewise the lines of the hours are made, 4
either the equal or the unequal ones,/ on the quad- 5
rant opposite to it, I mean the altitude quadrant,
since this is already encumbered with lines./ And 6
so, when the circles are described in it; one pre-
determines the altitude of each hour/ on it, and 7
if the [pointer][2] is then placed along that al-
titude then the edge of/ the alidade cuts that cir- 8
cle at the crossing of the line of that hour in it.

Indeed some of the astrolabe-makers make 9
lines on the alidade for the/ unequal hours, making 10
the operation for it to put the pointer of the ali-
dade along the equal of the/ noon altitude for the 11
[given][3] day. Then put the sun opposite the alti-
tude quadrant without/ moving the alidade from its 12
position, and observe skilfully the crossing of
the edge of the shadow of the upper sight/ at the 13
line passing through the middle of the alidade at
the length cut off for that unequal/ hour. And 14
thus is determined the (time) passed of the day or
the time remaining to it. As for the construction
of these/ lines, even though their drawing deviates 15
from true rigor, (nevertheless I will proceed to
describe it:) let there be supposed between the two
sights,/ the line passing through the middle of the 16
alidade, *TK*, (in Figure 48), and the two sights *TH*
(and) *KL*./ We extend *TH* and *LH*, and we describe 17
about center/ *H* and at any indefinite distance 18
which may befall, the quadrant *ZW*. And we divide
it/ into six equal parts. They are *ZA*, *AB*, *BG*, *GD*, 19

[1]Text شظية ; MS شظبه ; read شطبة.
[2]Text شظية ; read شطبة.
[3]Text المخطى ; read المعطى as in the MS.

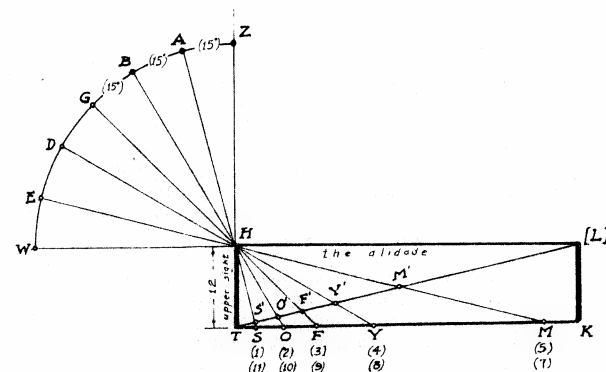



Figure 48

DE, and *EW*./ And we extend from them straight 189:1
lines all passing through center *H*. It is on/
the surface which follows the pole of the two sur- 2
faces of the sight. Then it is the shaded (part)
and not that which is/ along (the direction of) 3
the [pointer][1]. And these lines are *AHS*, *BHO*,
[*GH*]*F*[2], *DHY*,/ (and) *EHM*. Then we draw along// f.232b
the alidade lines lying across it perpendicular
to the line/ bisecting it lengthwise, passing 5
through the pole, (and) passing through points *S*,
O, *F*,/ *Y*, (and) *M*. As for the one passing through 6
S, it is the one marking the end of the first hour,
and so write/ the letter for one on one of its two 7
halves, and the letter for eleven on its/ other 8
half, because the (time) past and the (time) re-
maining of the hours are symmetric(ally disposed).
As for the one passing through/ *O*, it is for two 9
hours, so we write at it the letter for two on one
side, and the tenth/ on the other (side). And 10

[1]Text شظية ; read شطبة.
[2]Text دحى; read جحف.

(that) passing through *F* is (for) three hours, 189:10
and so we write at it a letter for/ three, and for 11
nine near it. The one passing through *Y* is for four
hours,/ and so we write at it a letter for four, and 12
eight, and the one passing through *M* is for five/
hours, and it will be lettered five and seven.

As for the sixth (hour), at it the upper 190:1
sight shades all of the lower sight,/ and hence the 2
letter for six is written above the orifice near
the upper edge/ in order to stick to the mark of *L*. 3
And verily we have finished the construction of the
lines of the hours on the alidade.

If we want to find points *S*, *O*, *F*, *Y*, and 4
M by a/ different operation we would divide the 5
sight *HT*, which is standing for the scale with its
digits, and *TK*/ is one of them standing for the shad- 6
ow, then we take from the table the tangent (*lit.*
shadow) of arc/ *WA*, which is seventy-five parts, be- 7
cause each one of the divisions of/ the quadrant is 8
fifteen parts, and we count that tangent (i.e. we
lay it off) from *T*, and we end/ at *S*. Then we take 9
the tangent of sixty, I mean arc *WB*. We count it
from/ *T*, and we end at *O*. Then we count the tan- 10
gent of forty-five from/ *T*, and we end at *F*. We 11
count the tangent of thirty from *T* and we termi-
nate (it)/ at *Y*, and the tangent of fifteen from *T* 12
to *M*. Ḥabash put/ these tangents in a separate 13
table, which is this:

Hours		The Shadow	
		Digits	Minutes
1	11	3	13
2	10	6	[5]6[1]
3	9	12	0
4	8	20	47
5	[7][2]	44	47
The Whole Alidade			

Figure 49

[1] Text ٯ ; read ق as in the MS
[2] Text و ; read ر .

(Due to) the known (fact) about the great 191:1
differences in the increments of the tangent (func-
tion),/ these lines are marked on instruments which 2
give us the hours without/ measurement of the alti- 3
tudes in another manner. The followers of this meth-
od join/ *T* (to) *L* and thus these lines made for 4
the hours will shift from *TK*/ to *TL* and the alidade 5
becomes *HTLK*. It is called the locust's thigh,
(since)/ it resembles it in form. And the pole of 6
it is made inside the instrument, underneath it./
For the most part the locust's thigh is made for the 7
instrument known as the moon's [crescent?][1]. The
two/ points *T* (and) *L* are not needed for the exten- 8
sion of line *AT* between them, but on the contrary it
may be/ extended from a point under *L*, or above it, 9
to a point under *T*/ or above it, and that depends 10
on the choice of the maker and his taste.

Among them (the makers) are those who trans- 11
gress the limits in the matter of simplification (by
using) an alidade called the crescent-like (one)/
since they make it in a semicircle like *ABG* (Fig.- 12
ure 50) with the base of the pole at *B*./ They di- 13
vide its interior into six equal parts// (for) f.233a
the hours, and they arrange it at the pole above/
the plane alidade in a manner which does not change 14
its position. Then they place the pointer along
the/ noon altitude, and they look along the concavi- 15
ty (*taḥdib*) of the crescent-like (alidade), and at
the position of the shadow of its edge as (in the)/
preceding. 16

Then they attach[2] to the astrolabe of this 192:1
type a plane sundial/ along a plane parallel to 2

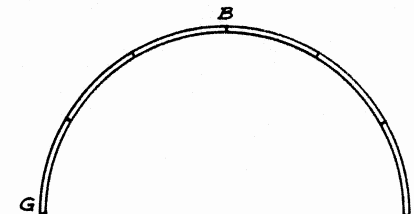



Figure 50

[1] Text بحق ; read محق ?
[2] Text ينقلون ; MS ينقلوا .

the horizon, and it is among the customs of the 192:2
craftsmen to work out the shadow of each/ hour and 3
its azimuth for the solstices. They work out the
amount of the shadow in digits of the sundial's
gnomon (*miqyās*)/ and its azimuth with its direction, 4
and thus they obtain the tips of the shadow for the
hours/ at the two solstices. 5

We have said that they will be on the peri- 6
phery of an hyperbola, and if one connects them
(the points)/ for each one of the two (conic) sec- 7
tions by curved lines, and (also connects) each of
the points of/ the (conic) section to its corres- 8
ponding (one) on the other, they will be the lines
of the hours. If one wants to construct the/ two 9
lines of the afternoon (prayer) on it; open the
compass by the amount of the digits of that line
at the time of the solstice,/ and describe about 10
the center of the gnomon (*miqyās*), and with a dis-
tance equal to that opening (an arc which) inter-
sects the (conic) section of that/ solstice at the 11
desired point. If the corresponding points in the
two sections are joined,/ [there will result][1] a 12
line for the two sections. But the types of the
fixed sundial are numerous, and the known (types)/
among them after the (horizontally) extended 193:1
(ones) are: those being in the plane of the me-
ridian, and those fixed/ in the plane of the prime 2
vertical, and those fixed in the plane of the celes-
tial equator. And if/ each one of these circles 3
were a horizon of a certain latitude, then the
lines of/ the hours and the lines of the times (of 4
prayer) are drawn on it in the fashion done with
the (horizontally) extended (sundial). As for/
that which is in the (plane of) the meridian, it 5
is on one of the horizons of ***sphera recta***.

As for that which is in the (plane of) the 6
prime vertical, it is on a horizon whose latitude
equals the/ complement of the local latitude, and 7
it is (well-)known that if the circle of the prime
vertical is rotated about/ that diameter (which is) 8
common to the horizon, and is inclined to the south
by the amount of the local latitude,/ it will 9

[1] Text في القطعين خطا ; MS في القطعين حصل خطا .

become the celestial equator at the horizon of a 193:9
position whose latitude will be a complete quadrant,
and hence the amount of the shadow of the gnomon/
will not differ in this sundial for any daily so- 10
lar path (***madār***); it/ will rather be equal to the 11
altitude (which is) equal to its declination. And
if it attains the shadow of the ***madār*** and its azi-
muth at/ the desired time of the hours and the af- 12
ternoon (prayer), and if (further) the same is done
with it as in the preceding (case) for the heads of
the signs/ for each one of its two faces; the nor- 13
thern, upper one, and the southern, the lower, and
if one joins the/ corresponding (points), there re- 14
sults the desired line. For the times of prayer,
instruments are made suspended/ by threads strung 15
through their extremities, parallel to the horizon,
like the ruler on which/ a gnomon (*miqyās*) is erec- 16
ted at need, and it is dispensed with when not need-
ed, so that it hangs/ with its surface level. 17
Verily they have made on it (the ruler) lines of the
times (of prayer) by their shadows/ according to 18
the days of the Byzantine months. (They are also)
like the plate of which half its diameter is equal
to the shadow of the end of/ the afternoon (prayer) 19
at the (time of) the winter solstice. Some people
divide its circumference into twelve for the (zo-
diacal) signs/ or the Byzantine months, and they 194:1
join the first (points) of them to the center by
straight lines[1],/ each one of which gives an esti- 2
mate of the shadows of the times (of prayer) or the
shadows of the hours. Then we join/ the corres- 3
ponding (ones) by arcs, and the resulting config-
uration looks like a citron, and it is named
after it.

Other people divide the circumference into 4
six parts, writing in them the signs of the ascen-
ding/ half (of the ecliptic) from right to left, and 5
near[2] them the descending half/ from left to right, 6

[1] Text مستقيمة من ويقدر ; MS مستقيمة ويقدر .
[2] Text قرانها ; MS قرائها .

and they do exactly as we explained, and there result the lines in the/ form of a [spiral][1] beginning from the first of Cancer to the first of Capricorn. (Now) what/ we have indicated suffices// for this subject, and about it are writings which treat of it exhaustively, by God's permission. 194:6 7 8 f. 233b

[1]Text الكوكب ; read اللولب as in the MS.

THE TWENTY-SEVENTH CHAPTER 194:9

ON THE USE OF THE SHADOW 10

IN THE QUADRILATERAL (MENELAOS) THEOREM

AND IN ASTRONOMICAL COMPUTATION

The practitioners of astrology simplified 11 much of what they found difficult/ to obtain from 12 astronomical arcs by replacing (them) by shadows, making concise a method (otherwise) long./ We 13 will refer to something of this (type) so that you may know how it is. Indeed there was a previous mention of/ ratios between sines which are equal 14 to ratios between the gnomon and its shadows. Since/ the people made the parts of the gnomon 15 equal to the parts of the total sine, they also made equal/ their two amounts, making of them the 16 radius of the circle. So there resulted/ from the 17 sines inside the circumference, polygons, and from the shadows outside it, (other) polygons/ similar 18 to the first. And so they were in proportion, since they were to one scale.

Let, for example, the two arcs *AB* (and) *AG* 19 (in Figure 51) be equal, and arc/ *BAG* measure 195:1 the circumference by a non-fractional number, and we extend *AET* and we take/ from center *E* an amount 2 *ET* equal to the gnomon, and we pass through the two points *A* (and)/ *T* the two perpendicular (lines), 3 *KH*, (and) *DZ* to *AT*, and we join to them *DBH* (and)/ *Z*[*G*]*K*[1], and we join *B* (to) *G*. And so *BG* will be 4 the side of a regular polygon inside/ the circum- 5 ference, and *DZ* is the side of a polygon outside the circumference (and) similar/ to the first. And 6 it is known that *TH* is the shadow of arc *AB*, re- versed (i.e., the tangent), and *KT*/ likewise is the 7 shadow of arc *AG*, reversed. If the gnomon is *ET*,/

[1]Text ح ; read ج .

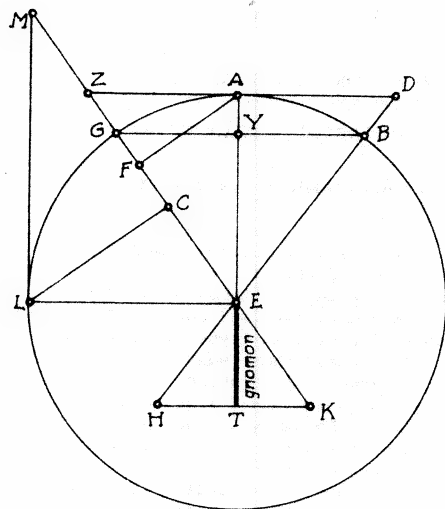



Figure 51

and the ratio of *BY* to [*Y*]*E*[1] is as the ratio of 195:8
TH to *TE*,/ and as the ratio of *DA* to *AE*, then 9
AD also is the shadow of arc *AB*, reversed./ If 10
the gnomon is *AE*, which is the total sine, then
the shadow *AD* is that which/ is in proportion with 11
the sines and not the shadow of *TE* because the
sines are of the type of the parts of/ *AE* and not 12
parts of *ET*. Likewise it can be shown that *AZ* is
the reversed shadow/ of arc *AG*, and shadow *DZ* is 13
composed of the reversed shadow of the two arcs/
AB (and) *AG*. And side *BG* is compounded of its two 14
sines,/ I mean *YB* (and) *GY*. Similarly had the in- 15
scribed polygon been compounded/ of multiples of 16
the sine of *AB*, which is equal to the sine *GY*, I

[1] Text به ; read ـه .

mean contrary to/ (our) situation, it will still 195:17
remain similar to the circumscribed (one) even
though it will no longer be parallel to it./ And 18
if the arcs are different, as are the two arcs *AG*
(and) *GL*, there will be compounded/ neither a poly- 19
gon of their shadows circumscribed about the cir-
cle, nor a polygon of their sines inscribed in it./
That is because the two sines *AF* (and) *LC* are 196:1
neither joined nor intersecting on the diameter/
EG at a single point. Similarly the two shadows 2
AZ (and) *LM* fail to intersect the/ diameter *EG* at 3
a single point, since this requires the equality
of the two arcs/ *AG* and *LG*. But it is known from 4
the situation of this picture that the reversed
shadow/ for each arc is what separates it from the 5
diameter passing through one of its two ends from
the line tangent/ to it at the other end, if we ex- 6
tend the two until they intersect./ The direct sha- 7
dow for it is what separates the diameter passing
through one of the two ends of its complement if it
is/ extended from the tangent line for it to the 8
other end. If one/ considers this as pertaining 9
to shadows we say that it is well established in
the book Almagest/ and in others that if between 10
two great circle arcs *AB* (and) *GB* (in Figure 52)
there intersect/ two great circle arcs *AD* (and) *GE* 11
at point *Z*, then the ratio of the sine of/ *EB* 12
to the sine of *AE* is compounded of the ratio of
the sine of *ZD* to the sine of/ *AZ* times the ratio 13
of the sine of *GB* to the sine of *DG*. So let us
assume the/ complete quadrilateral (*qiṭāʿ*) *ABG* 14
composed of great circle quadrants. It was
shown// in what preceded/ that the ratio of f.234a
the sine of each arc to its cosine is as the ra- 15
tio of its tangent/ to the gnomon. And so the 16
ratio of the sine of *EB* to the sine of *EA* is as
the ratio of the/ tangent of *EB* to the gnomon. 17
Likewise the ratio of the sine of arc *DZ*/ to the 18
sine of *ZA* is as the ratio of the tangent of *DZ*
to the gnomon. And the ratio of the/ tangent 19
of arc *EB* to the gnomon, hence, is compounded of
the ratio of the tangent of/ *DZ* to the gnomon 197:1
times the ratio of the sine of *GB* to the sine of/

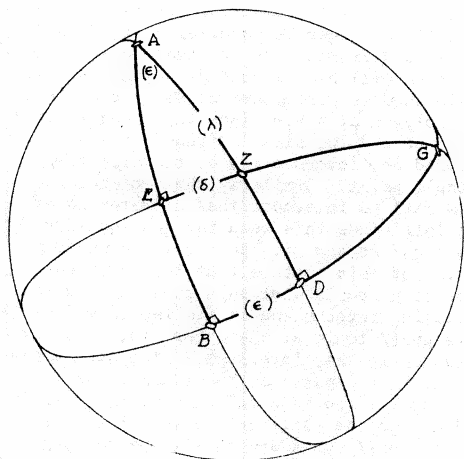



Figure 52

DG. But we put the gnomon equal to the total 197:2
sine, and arc *BG* is a/ quadrant of a circle, and 3
its sine is the total sine. And so the ratio of
the tangent of *EB*/ to the total sine is compounded 4
of the ratio of the shadow of *DZ* to the total sine
times the ratio of the/ sine of *GB*, the total sine, 5
to the sine of *GD*, and by inversion/ the ratio of 6
the total sine to the shadow of *EB* is compounded of
the ratio of the total sine to the/ shadow of *DZ* 7
times the ratio of the sine of *GD* to the total sine.

We have neglected the property of the (di- 8
rect) shadow because of the fact that the only one
we use of them is the/ reversed (one). And because 9
the first of these six amounts which make up this/
ratio is equal to the third, then upon dropping 10
them the ratio reduces from a compound (one) to/
the simple (one). That is that the ratio of the 11
first to the middle (one) between it and the second

is/ as the ratio of the third to the fourth. 197:12
But the first is equal to the third, and so the
middle (one)/ (above-)mentioned also is equal 198:1
to the fourth, and the ratio of this middle (one)
to the second is as the ratio of/ the fifth to 2
the sixth. But the middle (one) is to the fourth
as the ratio of the fourth/ to the second. And 3
this is the ratio remaining upon dropping the first.
So the ratio of the tangent of/ *DZ* to the tangent of 4
EB is as the ratio of the sine of *GD* to the sine
of/ *GB*. Therefore triangle *ZDG*, which is made up 5
of arcs of great circles, if there is/ in it a 6
right angle, like angle *ZDG*, then the ratio of the
tangent of one of the two sides/ bounding the right 7
angle to the sine of its other leg will be/ as the 8
ratio of the tangent of the angle opposite the first
side to the sine of/ the right angle, and it is the 9
total sine. I mean that the ratio of the tangent
of *ZD* to the sine of/ *DG* is as the ratio of the tan- 10
gent of angle *ZGD* to the sine of angle *ZGD*. [Thus
the ratio of the tangent of *DG* to the sine of *DZ*
is as the ratio of the tangent of angle *DZG* is
to the sine of angle *ZDG*.][1]

That which necessitates the use of the tan- 11
gent (function), [needless to say,][2] is that it is
restricted to/ the arc itself, and that the cotan- 12
gent is shifted to the complement of the arc.

No matter how we deviated from this in 199:1
what preceded which had to do with the reversed
(shadow) where/ the shadow and the gnomon only are 2
mentioned, and not the two appropriate sines, we
can make the numerator a denominator and the denom-
inator/ the numerator, interchanging the reversed 3
for the direct thus achieving harmony, and clari-
fying the matter of the/ direct shadow also, although 4
we do not use it.

However, as to how simplification occurs by 5
the use of the tangent/ for the extraction of ce- 6
lestial arcs, we return in discussing it to the

[1]Missing in the text; supplied from the MS:
وكذلك نسبت ظل دج الى جيب دز كنسبت ظل زاوية دزج الى جيب زاوية زدج

[2]Missing in the text: عني ما ذكر ; supplied from the MS.

preceding quadrilateral. 199:6
We say that if *AD* (in Figure 52) is the e- 7
cliptic, and it is supposed/ that arc *AZ*[1] is on it, 8
and in it we are required (to find) *ZE*, called the
first declination, then we multiply/ the sine of 9
this assumed arc by the sine of the inclination of
the ecliptic, and we divide the result by/ the to- 10
tal sine. There results the desired sine, because
the ratio of the sine of *AZ* to/ the sine of *ZE* is 11
as the ratio of the sine of *AD*, the quadrant, to the
sine of *DB*. However, if the/ ecliptic is *AB*, *EZ* 12
will be the second declination of arc/ *AE*. 13
Its determination from it is that we extend 14
the arcs of the quadrilateral along their circumfer-
ences in/ the two directions *A* (and) *G* (in Figure 15
53) until they intersect. And we describe from
pole *Z* and at a distance (equal to) the side of a
square (inscribed in a great circle)/ arc *TL*. And 16

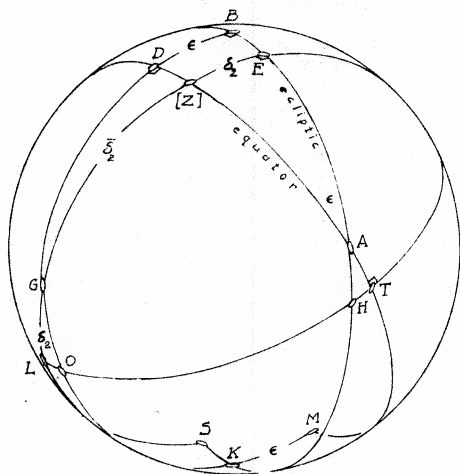



Figure 53

[1]In the text figure ز is missing.

so the ratio of the sine of *A*[*H*], the comple- 199:16
ment of *AE*, to the sine of *HT*/ // is as the f.234b
ratio of the sine of *AK*, the quadrant, to the 17
sine of *KM*, the inclination of the ecliptic./
And so *HT* is known, and the ratio of the sine 18
of *OH*, the complement of *HT*, to the sine of *OK*,
the/ complement of the inclination of the ecliptic, 19
is as the ratio of the sine of *HL*, the quadrant,
to the sine of *LS*,/ which is equal to the com- 200:1
plement of *EZ*. And so *EZ* is known.
Its computation is that we multiply the co- 2
sine of the [given][1] arc on the ecliptic/ by the 3
sine of the inclination of the ecliptic, and we di-
vide the result by the total sine, and of what
comes out we find its arc sine,/ and we subtract 4
its arc from ninety, and we divide the sine of
what remains into the product of the sine/ of the 5
inclination of the ecliptic by the total sine. Thus
there results the cosine of the desired second dec-
lination./ It cannot be obtained by the use of 6
sines except after two multiplications and two di-
visions and an extra arc (function) determination.
Whereas if we use the tangent for it it can 7
be obtained by a single multiplication and division
together with the elimination of/ that determina- 8
tion of the arc sine, because the ratio of the sine
of *AE* to the sine of *AB*, the quadrant,/ is as the 9
ratio of the tangent of *EZ* to the tangent of *BD*.
And so, if we multiply the sine of/ the [given][1] 10
arc on the ecliptic by the tangent of the inclina-
tion of the ecliptic, and we divide what results by/
the total sine, there results the tangent of the 11
second declination. If *AB* (in Figure 54) is the
celestial equator,/ and *AD* is one of the hori- 201:1
zons having (non-zero) latitude, and *Z* is the
rising point of a part on it, and/ *G* is the pole, 2
and *GB* is the meridian, *AE* would be the equation
of/ daylight of that part, and *ZE* its declination. 3
So if we are given *ZE* as the declination, and *GD*
as the/ latitude of [that horizon][2], and *AE*, the 4

[1]Text مخطاء; read معطاة.
[2]Text ذلك لا افق; read ذلك الافق as in the MS.

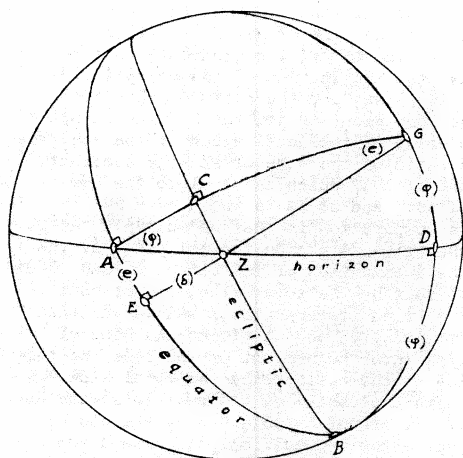



Figure 54

equation of daylight, is sought, then to use 201:4
sines/ we describe about pole *B* and at a dis- 5
tance equal to the side of a square (inscribed
in a great circle) arc *AG*. And we pass through/
the two points *B* and *Z* great circle arc *BZC*. 6
And so the ratio of the sine of *EZ*,/ the decli- 7
nation of the part, to the sine of *ZC*, is as the
ratio of the sine of *BD*, the complement of the
latitude,/ to the sine of *DG*, the latitude. So 8
ZC is known, and the ratio of the sine of *GZ*,/
the complement of the declination of the part, 9
to the sine of *ZC*, is as the ratio of the sine
of *GE*, the quadrant, to the/ sine of *EA*, the de- 10
sired (thing). So if we multiply the sine of the
declination of the part by the sine of the lati-
tude of the/ locality, and we divide what results 11
by the cosine of the latitude, then multiply what
results from/ the division by the total sine, and 12

we divide the result by the cosine of the dec- 201:12
lination of the part, there comes out the/ sine
of the equation of daylight. So the desired 13
(thing) results also by two multiplications and
two divisions.

When we multiply the tangent of the decli- 202:1
nation of the part by the total sine, and divide
the result by the/ tangent of the complement of 2
the local latitude, there results the sine of the
equation of daylight by (one) multiplication and
(one) division, because the/ ratio of the sine of 3
AE to the sine of *AB*, the quadrant, is as the ra-
tio of the tangent of *ZE* to the/ tangent of *DB*. 4
This amount of explanation should suffice, because
(to give) full due to its applications/in the 5
science of astronomy (*tanjīm*) would require an
exceedingly long time.

THE TWENTY-EIGHTH (CHAPTER) 202:6

ON THE DETERMINATION OF TERRESTRIAL DISTANCES 7
AND THE HEIGHTS OF MOUNTAINS
BY (THE USE OF) SHADOWS

We will take up, of these distances, those 8
which are limited and perpendicular, since they
are/ the shortest distances. The rest of them 9
are not limited in amount, except by circumvention.
The ray/ and the shadow have in common the indi- 10
cation of the one by the other. Illumination and
perception/ by eyesight have in common the prop- 11
erty of straightness. Hence there is no differ-
ence between/ operations valid for rays, shadows, 12
or visual perception, nevertheless we seek those in
which we use/ shadows. We say that these dis- 13
tances either are on the surface of the earth/,
or else they are above it or below it. 14

Those which are on the surface of the earth, 15
either they are from the observer,/ I mean that he 16
is on them, or else they are not// on his po- f.235a
sition. But this has/ nothing to do with what 17
we have, since operations are required different
from shadows. So, let the discussion be/ of the 18
first kind.

An example of it (is) the width of a val- 19
ley which it is desired to measure. And so, let
the investigator be stationed on a shore, and the
higher/ his position is, the more accurate will 203:1
his operation be. He sights through two holes in
the alidade of an astrolabe/ until he sees the 2
other shore opposite the two (holes) simultaneous-
ly, and he looks at the position of the pointer/
of the alidade in digits of the (horizontal) shad- 3
ow (i.e., cotangent), and he retains their num-
ber. Then let him move the alidade until/ these
digits are increased by one digit and it is left 4
at its position, and he backs up/ from his posi- 203:5
tion along the prolongation of the width which is
being measured, until a position is arrived at
such that it is seen/ in the (sight) holes as it 6
was seen at first on that shore. One measures the
distance between the two stations/ used, and it 7
is multiplied by the retained (amount). What re-
sults is the measure of the valley's width.

As for those which are above the surface of 8
the horizon, such as the height of a mountain, and
the positions of/ castles on it, and cupolas, and 9
pyramids, and minarets, if their summits are per-
ceived/ by sight, they are of two kinds. Either 10
the surveyor can reach the base of the height,/
I mean the point directly below it, or else he 11
cannot reach it.

As for the first kind, their shadows, if 12
surveyed at a time when/ the altitude of the sun 13
equals an eighth of a revolution, there will be
between the end of the shadow and/ the foot of the 14
vertical (a distance) equal to their heights.
If it happens that that altitude does not occur,/
put the pointer of the alidade at forty-five parts. 15
Then seek, by advancing and retiring,/ a position 16
from which the summit of the perpendicular is vis-
ible through both peep sights. And then one finds
the measure/ between the position and the base of 17
the perpendicular. We increase it by the amount
of the (observer's) [height][1] and there results/
the measure of the perpendicular. 18

The reason for this is evident, because of 19
the fact that the line of the ray or of sight bi-
sects/ the right angle formed by our bodies and the
line extending from them in the/ horizontal plane 204:2
to the foot of the perpendicular. If desired,
stand at any position one wants, like/ point *G* 3
(in Figure 55) on the earth, with the perpendicular
at *AB*. One should try to make/ *G* the position for 4
the center of the astrolabe by lying (prone) on
the ground or standing in a ditch/ as deep as one's 5
height. Then the astrolabe is suspended from the
right (hand) letting it hang with the quadrant/ of

[1] Text العامة ; read القامة .

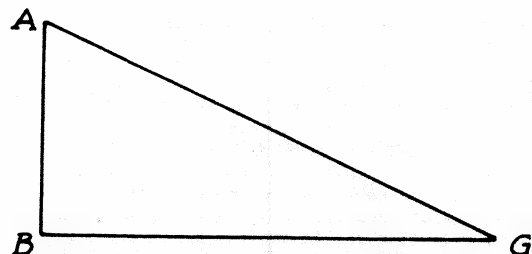



Figure 55

the altitude opposing the summit of the moun- 204:6
tain, and one looks through the two sights of the
alidade with one [eye][1] until it is/ seen through 7
both simultaneously, and one looks at the lower of
the two pointers (to see) how much of the shadow
is subtended, and one measures/ from that position 8
to the base of the vertical, I mean *GB*, and its ra-
tio to/ *AB* is known, because it is as the ratio of 9
that actual shadow to the gnomon,/ and hence if the 10
distance [*G*]*B* is multiplied by the gnomon and the
result divided by/ the actual shadow there results 11
the measure of the perpendicular *AB*.

Of this (type of) technique is what Brahma- 205:1
gupta explained in the arithmetical treatise of
the/ *Brahmasiddhānta* thus, "If a lamp is on a min- 2
aret whose length is a hundred/ digits; and in front 3
of it at a hundred and ten digits is a gnomon whose
amount is twelve/ digits, and we want (to find) the 4
amount of its shadow. So we multiply the hundred
and ten by twelve,/ and we divide the result by 5
eighty-eight, and there comes out fifteen and this
is the shadow of the gnomon".

So, let the minaret be *AB* (in Figure 56) 6
and the gnomon *GT* and its shadow *DG*,/ and we extend 7
TM parallel to *DB*. It will be a hundred and ten,
I mean/ equal to *GB*. And *AM* will be eighty-eight 8

[1] Text بين ;read بعين as in the MS.

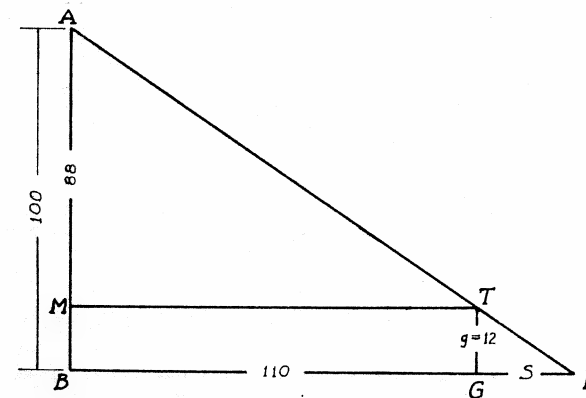



Figure 56

and the ratio of *TM* to/ *MA* will be as the ratio 205:9
of *DG*, the desired, to *GT*.

However, as for the second kind, in which 10
(the observer) does not reach its base, so he
measures from where he is,/ such as the perpen- 11
dicular *AB* (in Figure 57) being// inside the f.235b
mountain *ABG*. The nearest example of that where/
the foot of the vertical is inaccessible is (a 12
situation) where the sides of mountains or for-
tresses intervene between it and the surveyor./ So
let the flat ground which is in its vicinity be 13
GDE, and one should increase/ both *AB*, the per- 14
pendicular from the mountain, and *DB*, (the dis-
tance) between the position/ of the observer and 206:1
its base. So we measure the shadow of the alti-
tude of the summit *A* from station *D*/ as in the 2
preceding as to the conditions of the measurement,
and we retain it.

Then we retire or advance from that sta- 3
tion to another. Let it be *E* after/ that advance 4
or retreat, along the straight line joining the

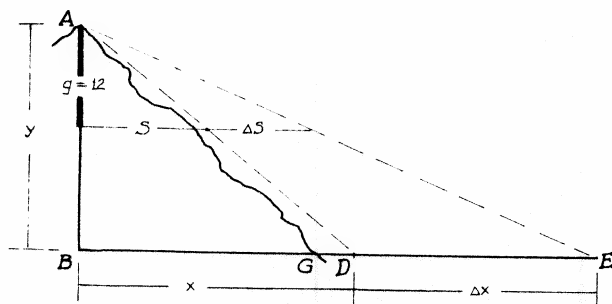



Figure 57

foot of the vertical/ from the summit and the 206:5
first station. And at this second (station) one
does/ what was done at the first, so that the 6
second shadow also is ascertained. And the dis-
tance between the two stations is measured/ in cu-
bits or in whatever linear unit is desired. What 7
results from that is multiplied/ by the divisions 8
of the gnomon, and the result of the multiplica-
tion is divided by the difference between the two
shadows,/ and there comes out the measure of the 9
height of the mountain in the units in which the
distance betwen the two stations was measured./
Then multiply also (the distance) between the two 10
stations by the length of the first shadow/ ob-
tained at position *D*, and we divide the result by 11
the difference between the two shadows,/ and there 12
comes out the distance from the first station to
the base of the vertical from the mountain (peak).
That (is)/ because the ratio of *ED*, taken as the 13
difference between the two shadows, to *AB*, taken
as/ the gnomon, is as the ratio of distance *ED* 14
to the distance *AB*, which is the gnomon./ Also 207:1
the ratio of *ED*, which is the difference between
the two shadows, to *DB*, which is/ the first shad- 2
ow, is as the ratio of distance *ED* to the dis-
tance *DB*. And that is/ what we wanted to clarify. 3

Of this (type of) technique is what Brah- 207:4
magupta explained in the (above-)mentioned book,/
thus, "If there is a lamp on a minaret, with an 5
obstacle intervening between us and its base, and
near/ [us][1] is a gnomon whose length is twelve dig- 6
its, and behind its shadow is another gnomon of
that size,/ its shadow being eighteen digits, and 7
from the head of the first shadow to the base of
the second gnomon is/ seven digits, and one [wants][2] 8
(to find) the length of the minaret. So the sec-
ond shadow is added to seven, and there results/
twenty-five, which is the base (for computation). 9
And we subtract the smaller of the two shadows from
the larger, and there remain (in)/ parts of the di- 10
vision (of the gnomon), three digits. Then the
base is multiplied by each of the two shadows./
[Divide][3] each one of the results by the parts of 11
the division, and there results (the distance)
from the/ base of the minaret to the end of that 12
shadow. And then we multiply by it the gnomon
and we divide what results/ by that shadow". There
results the length of the minaret *AB* (in Figure 13
58). (Let) the first gnomon be *TG*/ and its shadow 14
GD, and let the second gnomon be *HE*, and its shad-
ow *EZ*. We extend/ *HK* parallel to *AD*, and so *ZK* 15
will be the difference between the two shadows,/
and its (length is) the parts of the division. 16
As for the base, it is *ZD*, the sum of *ZE* 17
(and) *ED*, and from the similarity of the two tri-
angles/ *ZHE* (and) *ZAB*, the ratio of *BZ* to *ZE* will 18
be as the ratio of *EH*,/ the gnomon, to *AB*, the 19
minaret. And from the similarity of triangles
DTG (and) *DA*[*B*][4]/ the ratio of [*G*]*D*[5] to *D*[*B*][6] 208:1
will be as the ratio of *GT*, the gnomon, to *AB*,/
the minaret. And so the two ratios are equal, 2
and after substitution the ratio of *ZE*/ to *DG*, 3

---

[1] Text منها ; read منّا .
[2] Text يزيد ; read نريد as in the MS.
[3] Text ليقسّم ; MS يقسّم .
[4] Text ز ; read ب as in the MS.
[5] Text ب ; read ح .
[6] Text ج ; read ب

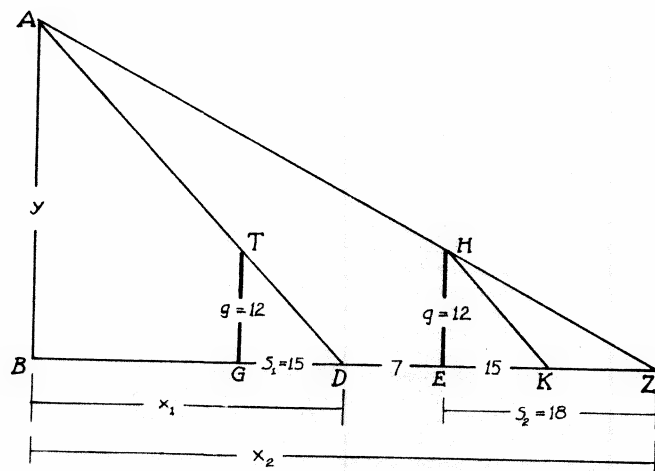



Figure 58

the two shadows, and the ratio of/ *ZK*, the dif- 208:4
ference between the two shadows, called the parts
of the division, will be by separation, to/ *DG*,
the smaller of the two (shadows, as *DZ* is to *DB*, 5
the smaller of the two) distances, and by inver-
sion, (*ZK* is) to *ZE*, the larger of the two shad-
ows,/ as the ratio of *DZ*, the base, is to *ZB*, 6
the larger of the two distances.

For that// which was explained there f.236a
are two other situations. The second gnomon is 7
set up,/ for the first of the two (cases), at the 8
end of the first shadow, (in Figure 59a), and so
the base contracts, and the divison of/ the two 9
shadows comes to be by its part, and the amount
of the second shadow will become accordingly, sev-
enteen/ digits and one part in twenty-two of a 10
digit. And in the other (case, Figure 59b) it
(the second gnomon) is set up/ on the first shadow 11

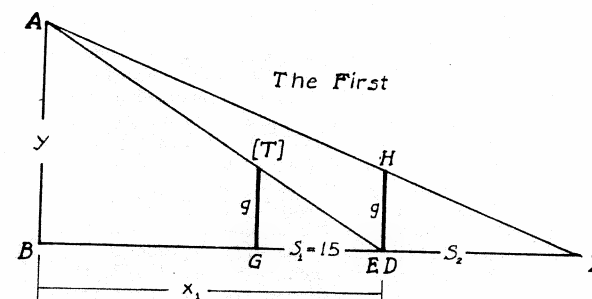
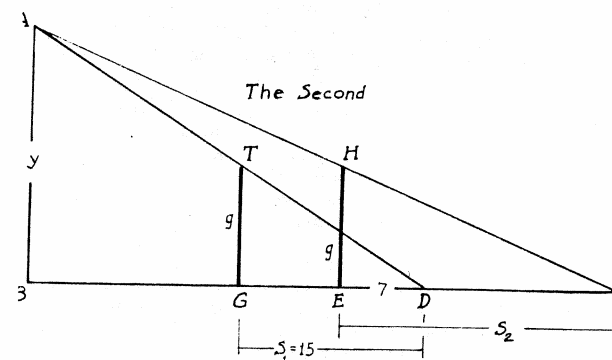



Figure 59a

Figure 59b

itself. Then the base will be the difference 208:11
between *ZE* (and) *DE*/ instead of the sum of the two 12
(which) was there (before). And if we assume *ED*
to be seven digits/ the shadow *EZ* would be nine dig-
its and (one) part in twelve of a digit, and this
is the/ picture of the two cases. 209:2

As to what is under the horizon plane, it 3
should be treated like what/ Brahmagupta explained. 4

If one imagines *B* (in Figure 58) to be the bot- 209:4
tom and *BG* the depth, and two sticks/ [*G*]*T*[1] and 5
EH, equal and parallel to the horizon plane, and
D (and) *Z*/ two stations for the observer (obser- 6
ving) *A*, which is a certain position assumed at
a deep place, and *B*,/ *E*, and *Z* are in order along 7
a straight line perpendicular to the horizon plane.
But if one measures position *A*/ at one of these 8
two positions, with the astrolabe, so that/ the 9
quadrant of the shadow is toward it, making the
second position such that the shadow at it will
differ by one digit, (then)/ *AB* will represent 10
the perpendicular from the mountain, and the depth
will represent the distance from its foot, and it
will be determined/ as was done previously. I 11
am planning to compose an exhaustive book as a
guide to the determination of distances which,/
I hope, will cover all its subjects and will con- 12
tain all that has reached me of the sayings about
it of/ the workers in this craft. 13

---

[1] Text ح ; read ج . In Figure 59a ط is missing in the text; supplied from the MS.

THE TWENTY-NINTH (CHAPTER) 210:1

ON CELESTIAL DISTANCES WHICH INVOLVE SHADOWS 2

On many occasions we do not confine ourselves 3
to the determination of distances of what is in the
world below, but we pass over/ to what the eyes per- 4
ceive in the upper world, especially if our guide to
it/ is its having a bright light which casts a shadow 5
for non-transparent objects. So let *AB*/(in Figure 60)
be the diameter of what appears of the sun's body, 6
and *ST* a plane surface opposite the sun/ and *EZ* a 7
body casting a shadow, placed higher than the face
of the earth, and the diameter of its observed shad-
ow is/ *HT*. And also let *EZ* itself be the diameter 8
of its hole. And we extend *BEM*./ So *M* will be the 9
end of the solar ray entering from the orifice *EZ*./
Let *L* be the midpoint of shadow *HT*. Whenever the 10
distances *LH*,/ *LM*, *EZ*, and *EK* become known to us, 11
the distance of the sun from the earth and its dia-
meter will become/ known also. 12

That is that triangle *EKM* with right angle *K* 13
will be known as to sides/, and we extend *EO* paral- 14
lel to *BT*, and we lay off *OT*/ equal to *EZ*, and there 15
remains *MO* known, and its ratio to *ME*/ is as the 16
ratio of *TM* to *MB*. And so *MB* is known and triangle
TMB is/ known as to sides, and the perpendicular 17
extending from *B* to *ST* is the distance of/ the sun, 18
and that is known, and the ratio of *TZ* to *ZF*, half
the difference between/ *HT* (and) *EZ*, is as the ratio 19
of *TB* to *BC*. And so *BC* is known/ and *AN* is equal 211:1
to it.

If there is added to the sum of [*C*]*B*[1] (and) 2
AN the amount *HT*, I mean/ *CN*, there results [*AB*][2], 3
and it is the diameter of the sun, and the situa-
tion is like it as to the distance of/ the moon and 4
its diameter, because if it (the moon) were [*G*]*D*[3]

---

[1] Text مب ; read صب as in the MS. In the figure, ر and ن have been restored from the MS.

[2] Text كل ; read اب .

[3] Text ح ; read ج .

Figure 60

the shadow of the body *EZ* (cast by) it would be/ 211:4
HT. But because of the fact that it is nearer to 5
the earth than the sun,/ its ray will enter into an 6

orifice [*EZ*][1] along *DES*, and the triangle *ESK* 211:6
replaces/ triangle *EMK* of the sun, and triangle 7
ESO of it replaces/ triangle *EMO* of it (the 8
sun).// The rest of the situations are as they f.230b
were. And also, both/ of the two triangles *EKM* 9
(and) *TFZ* are known as to angles, because their
sides/ are known. So triangle *MTB* is also known 10
as to angles, and has side/ *MT* known. So it is 11
known as to sides, and its perpendicular from *B*
upon/ the prolongation of *ST* is the desired dis- 12
tance, and what is between *T* and/ the foot of its 13
vertical added to *TL* is the radius of the sun.
But/ this will appear to be extremely diffi- [2]12:1[2]
cult if practised without understanding, some-
thing which causes the loss of confidence.

The method of Ptolemy for the determina- 2
tion of the solar distance uses the shadow also.
That is/ because the distance of the moon can be 3
obtained by parallax, which is not the case with
the sun,/ and the total solar eclipse does not 4
have a long duration in perception. He took, for
example,/ *AB* (in Figure 61) (as) the solar radius, 5
and *ZE* (as) the terrestrial radius. Let half of
the shadow/ cone be *ZET* [and][3] *OH* the lunar ra- 6
dius. So *HOE*/ is half the lunar shadow cone. 7
By lunar eclipses the shadow diameter had (pre-
viously) been obtained/ at the position of the 8
moon's transit (through the shadow). And so its
half, *DG*, is known, and the difference between
it and half the/ terrestrial diameter, which is 9
MZ, is known, and *DM*, the lunar distance, is
known. So triangle/ *ZMD* is known as to sides, 10
and triangle *ZET* is similar to it, and in it *ZE*/
is known. So it also is known as to sides. And 11
so *ET*, the distance of the end of the shadow
from/ the center of the earth is known, and the 12
ratio of *TE* to *EZ* is as the ratio of *TO*/ to *OK*. 13
And so *OK* is known. But *OH* for him is determi-
nable/ from lunar eclipses. So there remains *HK* 14

---
[1] MS هز ; missing in the text.
[2] Text ا ; read ٢ .
[3] Text ز ; read و .

Figure 61

as known, and we extend *KS*/ parallel to *HE*. So 211:15
it is equal to it, and hence will be known, and the
ratio of *ZS*,/ the difference between *ZE* (and) 16

[*K*]*H*[1], to *SK* is as the ratio of *ZE* to/ *EA*. And 211:17
so *EA*, the distance of the sun from the earth, is
known, approximately, and the ratio of/ *EH* to *HO*
is as the ratio of *EA* to *AB*, the/ solar radius,
and hence it is known.

We will go through the whole of the [2]13:1[2]
method of that in the promised book, and we
will solve what is connected with it/ as to am- 2
biguity and suspicion.

Sinān b. Fatḥ has put a chapter on the [2]13:3[2]
determination of the lunar distance from the earth,/
saying, "Observe the lunar shadow at meridian tran- 4
sit (***niṣf al-nahār***, usually "noon") and determine
its altitude from it./ Find it also by computation 5
and divide the total sine by the difference between
the two/ to obtain the lunar distance from the earth." 6
Taking into consideration what was previously men-
tioned concerning the lunar shadow's/ being dif- 7
ferent from the solar shadow with respect to the
gnomon, let, in such circumstances, circle/ *TDG* 8
(in Figure 62) be the meridian plane with the lu-
nar sphere, and circle *AB*/ (be) for the earth with 9
center *E*. And let *EG* be the true horizon plane,/
and *AD* parallel to it (and) tangent to the earth 10
at locality *A*. So if we assume *T* (to be)/ the
body of the moon, its altitude would be in the 11
meridian, either the apparent (altitude)/ *DT*, or 12
the computed (i.e. true altitude), arc *GT*. The
difference between the two is [*G*]*D*[3],/ and the ra- 13
tio of *DK*, its sine, to *DE*, taken as the total
sine, is as the ratio of/ *DK*, taken as one [2]14:1[2]
for its being equal to the radius *A*[*E*][4], to *DE*,
taken/ as the lunar distance in multiples of this 2
unit, and hence it is for that (reason) known.

The product of the total sine by the radius 3
of the earth is it (the terrestrial radius) exactly,/
and its quotient when divided by the sine of the 4
difference will be the desired distance.

[1]Text ع ; read ك as in the MS.
[2]Text ١ ; read ٢ .
[3]Text ح ; read ج .
[4]Text ا ; MS ه ا . In the figure, ح was supplied
from the MS. Text ن ; read ب .

Figure 62

It is as though there had dropped from Sinān's operations the mention of finding the sine, for it is a/ very apparent matter. — 214:5, 6

This is what Abū Yūsuf al-Kindī aimed at in a paper of his devoted to/ the lunar distance.//*f*.237a He took arc [*G*]*D*[1] as known, without mentioning anything/ about observing the altitude and computing it. He has nothing beyond Sinān except mention of the perpendicular/ in *Rūmī* (i.e. Greek), it being *qāthīṭ*. The method of Ptolemy for the extraction of the lunar distance/ at an assumed time and the extraction of its distance at other times is this (preceding). But/ it was taken as a fact to assume that there is no method for determining its distance except by its/ parallax, however, it is (also) possible by means of its eclipse in the shadow of the earth, of which (let) a half be *ABG*, (in Figure 63)/ — 7, 8, 9, 10, 11, 12, 13

[1] Text عد ; MS حد ; read جد.

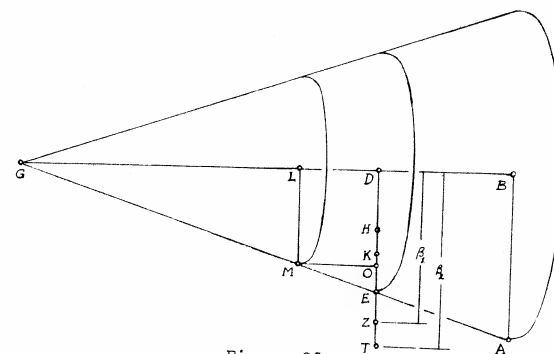



Figure 63

B being the center of the base. Let it (the eclipse) occur for an observer at a certain distance from/ the earth known in parts of the diameter of the inclined (orbital) plane. Let *BD* be a/ lunar eclipse of variable magnitude, the lunar latitude at it[2] (the time) being known, and let it (the latitude) be at/ circle *DT*. As for the eclipse occurring at *Z* with magnitude *EH*,/ let us assume it to be, for example, a third of the lunar diameter. However as for the one occurring at *T*, its magnitude (is) *EK*./ Let it be a fifth, by assumption. The (common) denominator of a third and a fifth is fifteen, and the difference/ between a fifth and a third is two (fifteenths), and the ratio of two to fifteen is as the ratio of/ *KH*, the difference between the two eclipses, which is equal to *TZ*, the difference between/ the two latitudes, to the apparent diameter of the moon at the known distance *BD*. *ZE*/ is known (so) a sixth of it (the lunar diameter) is known, and so the difference between this sixth and the latitude *ZD*,/ which is *ED*, I mean half the diameter of the shadow (is known). — [2]15:1[1], 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

[1] Text ١ ; read ٢.
[2] Text فيها ; MS فيهما.

Then (suppose) the same thing occurs at 215:12 another known distance. Let it be *BL*./ *ML* the shad- 13 ow radius, becomes known, and *DL*, the difference between the two distances/ *BD* (and) *BL* is [2]16:1[1] known, and its ratio to *OE*, the difference be- tween *ED* (and) *ML*,/ is as the ratio of [*G*]*D*[2] to 2 *DE*, and so [*G*]*D*[2] is known. And all of *BG* is known,/ and its ratio to *BA* is as the ratio of [*G*]*D*[2] to 3 *DE*. And so *AB* is known/ in parts of *BG*. Then if 4 *AB* is made a unit, the distances of the moon and the axis of the shadow/ cone in it (i.e. that unit) 5 will be known, and that is what we wanted to explain.

---

[1]Text ١ ; read ٢ .
[2]Text ح ; read ج .

THE THIRTIETH (CHAPTER) [2]16:6[1]

ON THE EXPLANATION OF THINGS CONNECTED WITH THE 7 SHADOW AND NOT RESEMBLING WHAT HAS PRECEDED

He who becomes acquainted with what is in 8 this chapter, and with the remaining (writings), which are unsound,/ will realize that there is nothing more 9 troublesome than (an attempt) to exhaust everything in this world. In the/ current problems with which the 10 Indian students are trained there is a long problem resembling/ what we are discussing. It is their say- 11 ing that if there is an umbrella (having a) diameter of four cubits; we desire/ to determine the distance 12 to which it should be elevated so that its shadow would disappear. Their answer is that we multiply the cubits of the/ diameter of that umbrella by a 13 quarter of an *ayuta* (transliterated as *ajūta*), and there will result the cubits of the desired distance for its elevation./ *Ayuta* is in their computations 14 ten thousand. It is as though multiplication/ would be by two thousand and five hundred, and ac- 15 cording to this it is necessary that the ratio of the/ solar diameter to the axis of the cone whose 16 vertex is the end of the earth's shadow be in the ratio of/ one to six hundred and twenty-five. 17

But (the value) which Ptolemy found for this 18 is the ratio of one to a hundred/ and thirty-four 19 approximately, because the solar distance from the earth according to him is a thousand/ and two [2]17:1[1] hundred and ten times the radius of the earth, and the axis of the shadow cone is/ two hundred and 2 sixty-eight times it, and the solar diameter is eleven times it. So, on the basis/ of Ptolemy's pa- 3 rameters it is necessary that the altitude of the umbrella be a hundred and/ thirty-four times its 4 diameter in order that its shadow disappear. But in their example it will be five hundred/ and 5 thirty-four cubits, as though the division had

---

[1]Text ١ ; read ٢ .

been dropped from their operation after multiplication. 217:5 But/ had it been (done) the (division) would have been nineteen approximately. The witness thereof, which we cited as being/ difficult to follow in an operation without imagination, is closer to that which Ptolemy has about it.// f.237b We constructed a/ target on a ruler five cubits long in order to consider what was previously mentioned in the chapter/ preceding this one, and we observed the shadow of the target on another one similar to it (placed) on the other end,/ (just) as we observed the light of the upper hole on a lower one, transforming the quantities into numbers/ which are integers and not fractions.

As for the numbers of the ruler (the distance) between the two targets is 6144, and the width of the/ target 164, and its shadow 116, which is [less by][1] 48, and therefore the vanishing of the shadow (occurs at) 20,992 from/ the target. And so, according to this ratio, if the solar diameter is eleven times/ the terrestrial radius, (the distance) from it (the sun) to the vanishing point of the earth's shadow will be 1408, of which the shadow has 256,/ leaving for the distance of the sun 11[5]2[2], [less by][1] 58 than the (number of) times mentioned by/ Ptolemy. Had it been, according to him, its mean distance, the (number of) times of its/ nearest distance would have been 1163, and what we found would be less by ten times. However, as for the/ number of the diameter, it is 18, and the number of its light is 59. So if the hole were equal/ [2]18:1[3] to the target its light would be five hundred and thirty-seven and five ninths, and if we/ converted all of the foregoing numbers into ninths to make them integers, the numbers of the ruler would become/ 55,296, and the number of the target 1476, and the number of its shadow 1044, and

[1]Text ينقصان ; read بنقصان.
[2]Text ١١٠٢ ; read ١١٥٢.
[3]Text ١ ; read ٢.

the number of the hole/ 218:4 equals that of the target at 1476, and the number of its light 4838. So the ratios of these numbers are/ known, and anyone who wants to use the quantities in them (may), since I do not find it worth wasting the time out/ of the best part of life, for verily I gave the amount of the ruler, it being in digits a hundred and/ twenty, and the radius of the earth in digits is approximately 321,563,636.

Let us move on from it to the determination of the solar distance, and from it to a lunar distance/ in a situation of total eclipse with zero duration of totality, so that we obtain for the moon what corresponds to/ what was obtained for the sun.

Among the things pertaining to this chapter is that the solar distance is continuously changing/ between its two limits, the maximum at the apogee and the minimum at the perigee, so that the axis of the/ shadow cone and the base of the shadow become smaller and greater. To the amount of light and shade on the face of/ the earth, al-Fazārī refers in his statement in his *zīj*, "Since the sun is larger/ than the earth, that which remains of it (illuminated) is more than half of it".

"So, if you want to determine the excess of that over half the earth, multiply the minutes of/ half the orb (*falak*) of the sun by the number of ***farsakhs*** in the circumference of the earth, which is 6583,/ and divide the result by 21600; there will result the number of ***farsakhs*** by which the light exceeds/ half the earth on that day".

The explanation of this operation is, [2]19:1[1] let the orb of the sun be *ABG* (in Figure 64) with/ center *E*, and the circle of the earth is *HTM*, and we assume both/ *AB* (and) *AG* to be to the amount of half the solar orb, that is, its diameter, and we extend *BTZ* (and)/ *GMZ*. So the axis of the cone will be *AEZ*, and we extend the earth's diameter/ ***DEK*** perpendicular to the axis, and we connect *E* (with) *T*,(and) *E* (with) *M* between the

[1]Text ١ ; read ٢.

Figure 64

center/ and the two points of tangency. So the 219:6
diameter of the shadow base will be the chord
TM. And because/ the two angles *ZTE* (and) *DE*[*Z*][1] 7
are right (angles) the two triangles will be
similar, and the two angles *TZE*/ (and) *DET* are 8
equal. The arc *DT* is to the amount of the angle
DET, and the angle/ *TZE* is [half][2] the arc simi- 9
lar to it on the circle described with center/ *E* 10
and radius// *EZ*, and that arc is *LS*. We ex- f.238a
tend *ELO*, and/ *AO* will be what is similar to *LS* 11
on the solar orbit. But al-Fazārī (made) its
place/ *AB*, and so the ratio of its minutes to the 12

[1]Text ج ; read ز . In the figure, س ز د ح ه ا and ع are missing or wrong in the text; they were restored from the MS or the context.
[2]Text ضعف ; read نصف .

minutes of a whole revolution equals the ratio 219:12
of the/ double of *DT*, I mean the sum of *DT* (and) 13
[*K*]*M*[1], to the circumference of the earth. But/
DHK is half a circumference, so the sum of the 14
two (above-)mentioned arcs is the excess of the/
lighted segment over the shadowed (segment). 15

However, as for the (above-)mentioned [2]20:1[2]
farsakhs, it is necessary to notice that the In-
dians measure/ distances by a length taken as a 2
unit which they call a *yojana*. Its magnitude in
our units is two and/ two thirds ***farsakhs***, so 3
that the cubits in each *yojana* are thirty-eight
thousand./ Measured in another unit of theirs 4
called the ***kroh***, there are eight in it (the *yo-
jana*), and each ***kroh***/ equals one of our miles. 5

Brahmagupta claims that the circumference 6
of the earth in *yojanas* is five/ thousand, and 7
its diameter is a thousand five hundred and eighty-
one approximately.

But Pulisa claims that the diameter of the 8
earth in them is a thousand and six hundred, and its
circumference/ five thousand and twenty six. 9

An explanation of what al-Fazārī mentions 10
as to the ***farsakhs*** of the earth, it being dif-
ferent from both (the other)/ assertions, is that 11
he heard and adopted the statement of Pulisa.
Then he learned and worked out, as is mentioned
in his zīj,/ that the Indian ***farsakh*** is sixteen 12
thousand cubits. Then he wanted to find the num-
ber of *yojanas*/ in a twelve-thousand cubit ***far-
sakh***, with its being less than the first by a 13
quarter of it,/ so he added to what Pulisa mentioned 14
a quarter of it, which is 1256, and so there result-
ed for him what he mentioned of/ the ***farsakhs*** in 15
the circumference without any investigation of
the writings of the (other) people. But at any
rate he is/ nearer to the truth than others like 16
him who have heard, as much as he, of the name of
the Almagest,/ but never dealt with any part of it. 17
Thus some claim that it was summarized from the

[1]Text طم ; read كم .
[2]Text ١ ; read ٢ .

Sindhind, and others set up/ computations like 220:18
the babblings of epileptics, ascribing them to it
(the Almagest). I have seen a **zīj**, the name of/
the author not being mentioned, which includes 19
this operation for the determination of the solar
hoop,/ from the Almagest, it is claimed. [2]21:1[1]

So it says, "Add to the square of the shad- 2
ow a hundred and forty-four and take the (square)
root of/ the result, and it will be the hypotenuse 3
of the shadow for the time (in question), and di-
vide 41,256 by it to obtain the minutes of the/
solar hoop. And, if desired, make the solar dis- 4
tance a versed sine in minutes of/ the chord, 5
which is 3,438. Subtract it from it, and there
will remain the vertical at that hour. If de-
sired, make the/ altitude at the hour a sine in 6
minutes of the chord, and it will be the solar hoop.
Then multiply it/ by twenty-three and divide what 7
results by sixty, and there will come out the ver-
tical at that hour. By it/ the difference between 8
zījes and dates is made known".

As for the first of his operations, it is 9
evident from what has preceded that the product
of/ the gnomon by the total sine, if it is divi- 10
ded by the cosecant for the time (in question),
there comes out the sine of the/ altitude of the 11
sun at the time of the observation, and this is
what we divided, and it is the product of/ twelve 12
times 3438 minutes, the total sine according to
Āryabhaṭa. He took it/ according to the ratio of 13
the diameter to the circumference. So that that
which he calls the solar hoop is the sine of/ the 14
altitude at the hour (in question).

As for the second operation, the "solar dis- 15
tance" in it is its declination, and the "minutes
of the chord"/ is the total sine, and the differ- 16
ence between it and the versed sine for the dec-
lination,/ is the cosine of the declination, I mean 17
the radius of the sun's daily circle. The name
"solar hoop"/ for it is more legitimate, and the 18
name "vertical at the hour" for the sine of the al-
titude is more legitimate.

[1] Text ١ ; read ٢ .

As for the third operation, it is nothing 221:19
but the transformation of the sine of the altitude
at the hour/ from the amount 150 to that [2]22:1[1]
which is found with Āryabhaṭa. But it is trans-
formed by what he mentioned into/ 3450; an amount 2
differing (both) from the total sines of Āryabhaṭa
and of Brahmagupta, for/ with him (i.e. Brahmagupta) 3
it is 3270. That is that the hundred and fifty
does not number the three thousand/ four hundred 4
and thirty-eight twenty-three times, but rather that
it numbers it/ twenty-two// times and twenty- f.238b
three parts of a twenty-fifth of a time. 5

Thereupon he said, "An example is that we 6
want to (determine) the difference between two
zījes, the Sindhind (and)/ the Shāh. So, because 7
the Sindhind is based on the Cupola, its longitude
being ninety, and its chord/ a hundred and fifty, 8
we multiply it by twenty-three and divide the result
by sixty./ There results [57];30[2], which we retain. 9
And because the Shahriyārān is based on Babylon/
at a longitude of seventy-eight and a latitude of 10
thirty-six, which is (in) the fourth climate, and
the/ (meridian) altitude of Aries at it is [5]4[3] 11
and its chord (is) 122, we multiply it by 23 and
divide/ the result by sixty, and there results 46, 12
46. We take the difference between it and the re-
tained (amount)/ and it is 10,44. We find its arc 13
by multiplying it by eleven, and divide the result/
by seven to obtain 16,52, which we make a chord, 14
it being [0];43,16[4]./ We set it aside, then we 15
make the latitude of Babylon (into) hours by di-
viding by fifteen. There comes out/ [two][5] hours 16
and two fifths; the sun travels in it [0],5;[55][6].
We added (it) to the (quantity) set aside;/ there 17
results [0],49[7]. We make the distance of Babylon

[1] Text ١ ; read ٢ .
[2] Text بـزل ; read نزل .
[3] Text بد ; read ند .
[4] Text ه ; read ة .
[5] Text ساعة ; read ساعتين .
[6] Text ههك ; MS ةهك ; read ةهنه .
[7] Text ه ; read ة as in the MS.

from the Cupola in hours; it will be four/ 222:18
fifths of an hour, in which the sun travels
[0];1,[57],36[1]. We add (the amounts) of the two
(distances) travelled; there/ results [0;51][2], and
that is (what is) between the two zījes".

It is evident that he meant to transform [2]23:1[3]
the sine of the altitude of Aries for each/ of the 2
two locations from the sine of two and a half parts
to the sine of/ fifty-seven and a half parts in or- 3
der that we obtain the difference between the two
(localities).

What is after that is words without meaning, 4
since the difference is between the mean (positions)/
if it is according to the meridians, and the lati- 5
tudes do not enter here.

But if it were according to the horizons its 6
amount would not be fixed in parts having/ one direc- 7
tion. It will be different in the two directions,
positively or negatively, and there is no use in
what/ was explained, and nothing can be deduced from 8
it. At least he could have asked where Babylon is,/
so as not to put it in the fourth climate, and not 9
carry it from Baghdād to Nīshāpūr./ If it were not 10
that the majority in all professions are like this,
then it would not have been that (only) a few deserve/
praise and adulation. Astrology is characterised 11
by abundance of these qualities, and the apportion-
ing of fates is/ more appropriate for it. 12

If you aspire to witness the truth of that, 13
look at the place of Māshā'allāh among/ the people, 14
and listen to his presumptuous criticism of the
book ascribed to Hermes,/ "The Eighty-five Chapters" 15
(*Al-Khamsa w'al-thamānīn bāb*) in order to be intro-
duced to it. Then turn to the book, and see which
of its contents beguile you/ in solitude and save 16
you from ending in chains in asylums in case/ your 17
(astrological) temperament is equable and your judge-
ment sound.

An example of that is the years of the planets 18

---

1 Text ه ا يز لو ; read ه ا ز لو .
2 Text يا ه ; read نا ه .
3 Text ١ ; read ٢ .

in it; these are assumed numbers/ for each one 223:19
of them, for Saturn thirty-two, and for Jupiter
the double of that, and for Mars/ equal to [2]24:1[1]
one and a half of it, and for the sun equal to
one and a half that of Mars, and for Venus equal
to one/ and a quarter (times) that of Mars, and for 2
Mercury fifty-one, and for the moon thirty-three./
It may be that they vary because of differences in 3
the copies. But there is no use in that, since the
importance is only/ of what comes after. Verily it 4
was said in it that they are put for the middle of
the earth and for the countries which adjoin the/
northern axis. So they modify them for countries 5
in which nativities occur according to their dis-
tances/ from the northern axis, and the equation 6
is subtracted from it if it is nearer the east,/
but they add it to it if it is nearer the west. 7
The maximum longitude is a hundred and eighty/
parts, and the northern axis is along the ninetieth 8
of them. So he asserted the use of the equation
for countries/ according to the mean (motions) of 9
the planets if the position (is reckoned) accord-
ing to the Cupola. Then he contradicted this in
what follows/ that concerning the extraction of 10
the equation. That is that he prescribed the sub-
traction of the altitude of the beginning of/
Cancer (at) noon from ninety degrees, and the mul- 11
tiplication of the remainder by a hundred/ and 12
fifty, and the division of// the result by 360. f.239a
There results a sine; find its arc (sine), and if
the equatorial/ shadow for the locality is more 13
than seven digits, subtract what resulted/ for
the arc from the years of each planet, but if it 14
was less add it, and they will be (thus) modified/
for the assumed locality. These seven digits men- 15
tioned in the condition of the equatorial (shadow),/
as well as[2] in the equation of the ascensional dif- 16
ference in the Arkand Zīj originate from/ one tem- 17
perament or two related temperaments. But the
distance from the meridian circle for the Cupola
does not/ move the azimuth of the pole in the di- 18
rection of east or west.

---

1 Text ١ ; read ٢ .
2 Text هاهى ; MS هاصى .

(Even) that is better than[1] those who 224:19
believe in the flatness of the earth/ and in [2]25:1[2]
the parallelism of vertical (lines), which belongs
to confused information,/ for concerning noon they 2
hold self-contradictory (opinions) to the extent
that some of them are of the opinion that the
time of noon/ is the same in all inhabited places. 3
Thus they base themselves on/ false premises, which 4
entail as a result their deviating in prayer away
from the/ true direction. 5

Some of them carry to an absurdity the af- 6
firmation of the practitioners of this science
concerning the difference/ in noon(time) at (dif- 7
ferent) localities, applying it to (distances)
less than ten paces. Such a one is (the fellow)
called/ Aḥmad b. Salmān with his saying, "One 8
way to determine noon while explaining/ the re- 9
sult at the same time is to take two rods, equal
in length and width,/ and to set up one of the two 10
along the direction of prayer and the other to the
left of it, and observe the shadows of both,/ and 11
if the shadow of the former is greater than that
of the latter, the sun has reached the meridian,/
but if the shadow of the latter is greater it has 12
not culminated yet".

I think that the author of these words has 13
done nothing save observing their shadows (cast
by) the light of/ a lamp which is not far from 14
either of them. This will be the situation of one
who goes out of a house/ through the roof and not 15
through the door. The book "The Eighty-five
(Chapters of Hermes)" is/ followed by (another) 16
book like it in which was mentioned the equation
of the degree of the ascendant in (connection with)
the rising amplitudes/ for the locality, if one 17
wants to use them for the determination of (people's)
ages. It is that one adds its declination/ to the 18
complement of the latitude of the locality if it
is northerly, and subtracts from it if southerly,/
and a quarter of the result is found. If the 19

[1] Text مسن ; MS من .
[2] Text ٩ ; read ٢ .

equatorial shadow for the locality is less 225:19
than seven/ digits, subtract that quarter from [2]26:1[1]
the degree of the ascendant, and if it is greater
add/ the quarter to it to obtain the degree of 2
the ascendant.

It is one of the marvelous things. But I 3
do not say this to slander Hermes, for he was so/
wise that the Greeks counted him a prophet. He 4
introduced/ Chaldean science into Egypt. The 5
Chaldeans were the people of Babylon, whose share
in/ science cannot be concealed, to the extent 6
that they were called its sorcerers, even though
nothing came down to us of their science except
their opinion/ concerning the motion of the heaven 7
which is based on a continuous solicitude in ob-
serving it for thousands of years,/ (together with) 8
what the observers, Ptolemy and the others, relate
concerning them. But/ in the books of alchemy and 9
talismans there is a serious fallacy,/ which is 10
the setting of charlatans to make them. Imita-
tion of these books is more prevalent in the case
of the wiser and the older of them,/ because of 11
the hidden character of the information, due to
its antiquity. And also the one branded with the
unravelling of secrets is more subject to it due
to the/ conjoining of their words with enigmas 12
and symbols. Now I suppose that this amount of
information/ about matters concerning shadows 13
should suffice and be/ helpful in the verifica- 14
tion of time (as determined) with instruments by
shadows.

God, be He exalted!, is the Helper, and the 15
Praised at the beginning of each treatise and at
its end. By the praise of God and His help,/
finished is "The Exhaustive Treatise on Shadows",/ 16
the work of Abū al-Rayḥān Muḥammad b. Aḥmad al- 17
Bīrūnī, may God forgive him.

I finished copying it at Mosul (Mawṣil) in 18
Dhū al-Ḥijja of the year 631,/ and to God be the 19
praise, and the prayers of God (be) upon Muhammad
and his relatives.

[1] Text ١ ; read ٢ .